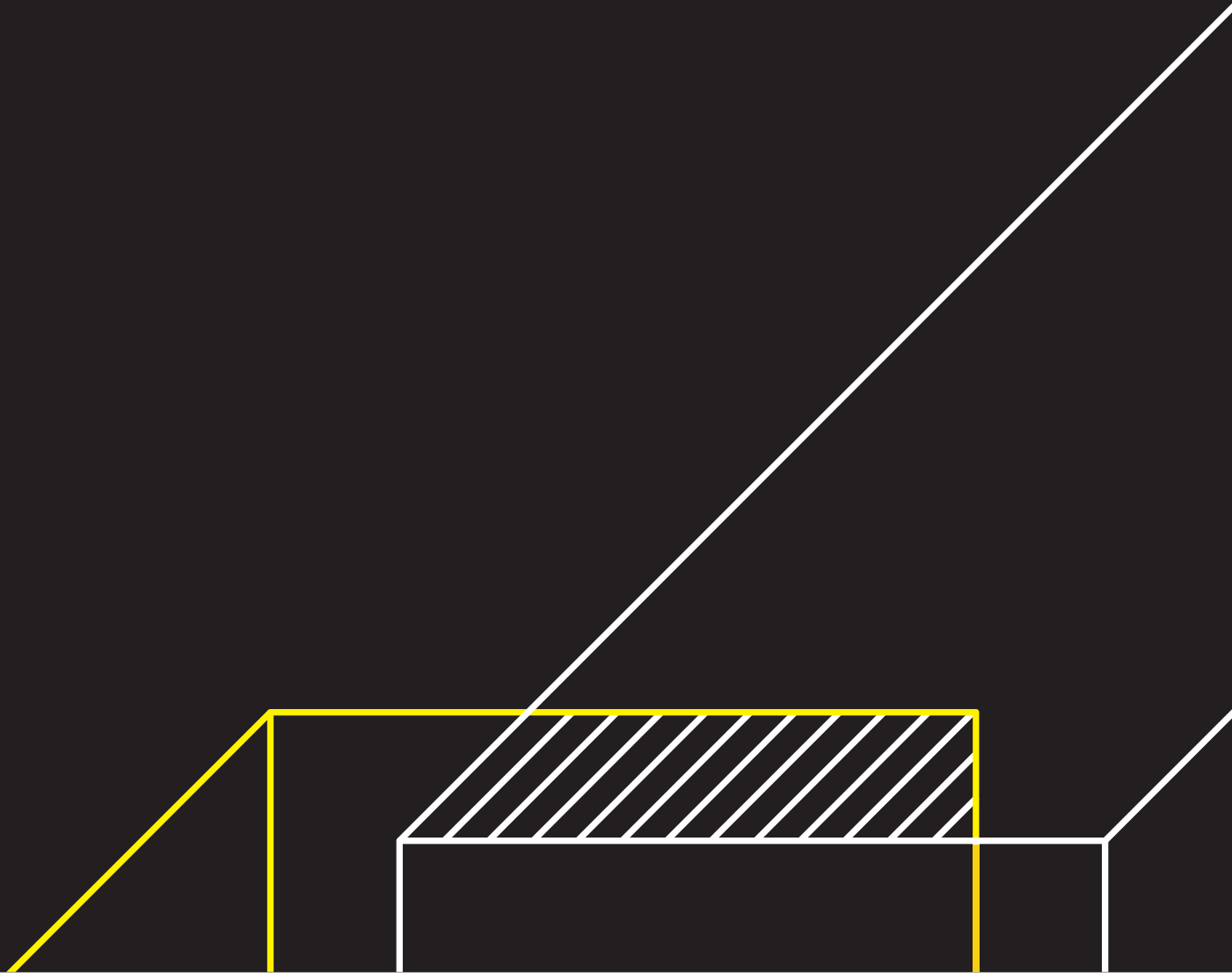


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НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ УНИВЕРСИТЕТ

PROCEEDINGS
OF THE PME
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RUSSIAN
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**TECHNOLOGY
AND PSYCHOLOGY
FOR MATHEMATICS
EDUCATION**

Moscow, 2019



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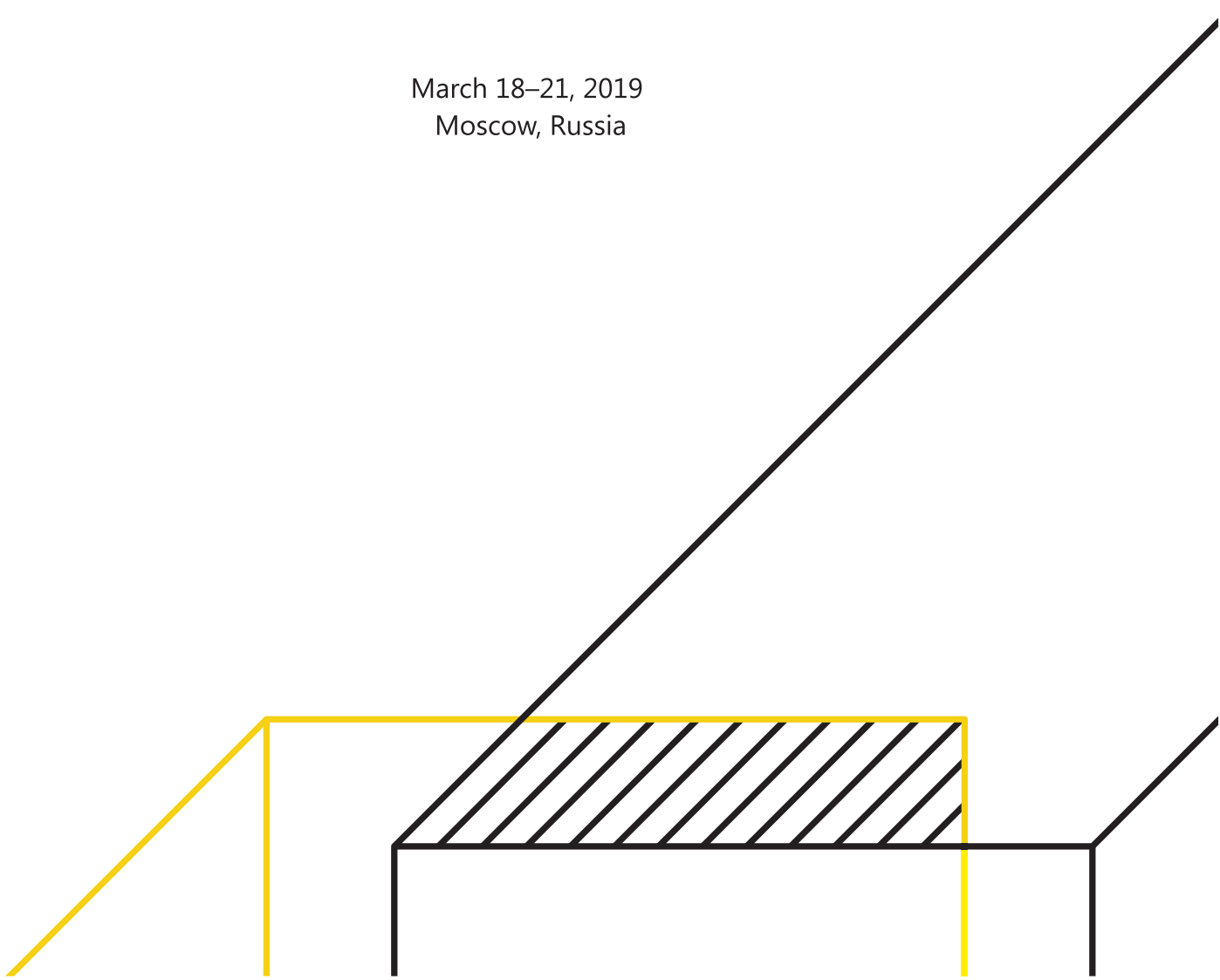
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Plenary Lectures, Plenary Discussions,
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EXPLORING FRIEZES AND ROSETTES: AN EXPERIENCE WITH FUTURE TEACHERS

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This study, part of an ongoing project, analyses the performance of future teachers of primary education (3–12 years old) in identifying and constructing symmetries, especially friezes and rosettes, with different resources (paper/pencil; software).

Geometric transformations are one of the most important applications of mathematics in daily life, allowing the establishment of rich connections. It enables students to explore/create patterns, solve problems and think spatially. However, students generally show a low level of learning when geometric transformations are concerned (e.g., [Swoboda, Vighi, 2016]). Research supports the assertion that appropriate technology and media-supported instruction can help with learning in a variety of domains, including geometric transformations [NCTM, 2014]. So, technological tools may be useful to develop visual skills and overcome some difficulties.

To conduct this exploratory study we followed a qualitative approach. The participants were 14 future teachers of primary education. Data was collected during the classes of a Didactics of Mathematics course, through: observation, written productions and photos. They were exposed to the teaching of geometric transformations (translations, rotations, reflections and glide reflections), analysing examples of applications in mathematics and other areas. The students were also motivated to identify/construct friezes and rosettes, using paper and pencil. After a period of appropriation of the processes involved, they were invited to explore the same aspects in a dynamic environment provided by the software *Gecla*. The functions used were: *Search for symmetries*, *Classify/Generate Friezes and Rosettes*.

Preliminary results show that these students easily identify symmetries with both resources and are comfortable with the construction/generation of friezes and rosettes, since it is a step-by-step process. These students exhibited difficulties in identifying the motif/module that generates some friezes/rosettes. *Gecla* allowed them to develop an intuition in some of these cases and increased their motivation, however certain students referred that the software could show the composed motif besides the minimum motif.

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