ETEN

Journal of the European Teacher Education Network

JETEN

VOLUME 4

ETEN Conference 2008

Journal of the European Teacher Education Network (JETEN)

VOLUME 4

Edited by **Kristiina Kumpulainen** and **Auli Toom**

ETEN Conference 2008

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First published in 2009 by ETEN and the University of Helsinki Department of Applied Sciences of Education and CICERO Learning University of Helsinki
P.O.BOX 9 (Siltavuorenpenger 20R)
FIN-00014 University of Helsinki
Finland

ISBN 978-952-10-4974-3 (nid)
ISBN 978-952-10-4975-0 (pdf)

Design and layout by Päivi Talonpoika-Ukkonen, Yliopistopaino

Subscription:
The annual subscription to the journal is charged through the conference registration fee, so an individual registering and attending the annual conference can expect to receive the volume of the journal following the conference. Any person or institutions may apply for an annual subscription to the journal.

Contents

The influence of visual strategies in generalization: a study with 6th grade students solving a pattern task

Ana Barbosa, Isabel Vale and Pedro Palhares

Abstract

This paper gives a description of an ongoing study focused on pattern exploration and generalization tasks. It evaluates the ways in which visual strategies can be used to enhance and enrich learners' experiences of generalization. The main purpose is to analyse strategies and difficulties presented by grade 6 students when solving these activities and to ascertain the role played by visualization in their reasoning.

Keywords: patterns, visualization, generalization strategies, difficulties.

Introduction

A quarter of a century ago, problem solving became a focus of school mathematics, and still is presently. According to the most recent curricular guidelines of several countries, one of the main purposes of mathematics learning is the development of the ability to solve problems. In spite of the curricular relevance given to this theme, several international studies (SIAEP, TIMSS, PISA) have shown that Portuguese students perform badly when solving problems (Ramalho, 1994; Amaro, Cardoso & Reis, 1994; OCDE, 2004). Pattern exploration tasks may contribute to the development of abilities related to problem solving, through emphasising the analysis of particular cases, organizing data in a systematic way, conjecturing and generalizing. For instance the *Principles and Standards for School Mathematics* (NCTM, 2000) acknowledges the importance of working with numeric, geometric and pictorial patterns. This document states that instructional mathematics programs should enable students, from pre-kindergarten to grade 12, to engage in activities involving understanding patterns, relations and functions. Besides, work with patterns may be helpful in building a more positive and meaningful image of mathematics and contribute to the development of several skills (Vale, Palhares, Cabrita & Borralho, 2006). On the other hand, Geometry is considered a source of interesting problems that can help students develop abilities such as visualization, reasoning and argumentation. Visualization, in particular, is an important mathematical ability but, according to some studies, its role hasn't always been emphasized in students' mathematical experiences (Healy & Hoyles, 1996; Presmeg, 2006). Although the usefulness of visualization is being recognized by many mathematics educators, in Portuguese classrooms teachers privilege numeric aspects over geometric ones (Vale & Pimentel, 2005). Considering it all, we think that more research is still necessary concerning the role images play in the understanding of mathematical concepts and particularly in problem solving.

This study intends to analyse difficulties and strategies emerging from the work of grade 6 students (11-12 years old) when solving problems involving patterns, and the role played by visualization on their reasoning. The tasks used in the study require pattern generalization. Students of this age have not yet had formal algebra instruction, thus the importance of analysing the nature of their approaches. This study attempts to address the following research questions:

1. Which difficulties do 6^{th} grade students present when solving pattern exploration tasks?
2. How can we characterize students' strategies?
3. What's the role played by visualization on students' reasoning?

Patterns in the teaching and learning of mathematics

Many mathematicians share an enthusiastic view about the role of patterns in mathematics, some even consider mathematics as being the science of patterns (Steen, 1990). The search for patterns is seen by some as a way of approaching Algebra since it is a fundamental step for establishing generalization, which is the essence of mathematics (Mason, Johnston-Wilder & Graham, 2005; Orton & Orton, 1999; Zazkis & Liljedahl, 2002). Searching for patterns in different contexts, using and understanding symbols and variables that represent patterns and generalizing are significant components of the mathematics curriculum in many countries. Portuguese curriculum mentions the importance of developing abilities like searching and exploring numeric and geometric patterns, as well as solving problems, looking for regularities, conjecturing and generalizing (DEB, 2001). These abilities are directly related to algebraic thinking and support the development of mathematical reasoning. Pattern seeking also plays an important role in the development of mathematical reasoning and in connecting mathematical ideas (NCTM, 2000).

The nature of mathematical thinking

Patterning activities can be developed in a variety of contexts (numeric, geometric, pictorial) and through the application of different approaches. Gardner (1993) claims that some individuals recognize regularities spatially or visually, while others notice them logically or analytically. In fact, it is common, in mathematical activities, that different individuals process information in different ways. Many students favour analytic methods while others have a tendency to reason visually. A study developed by Krutetskii (1976) with a sample of mathematically gifted students showed that they used different approaches in problem solving. While analysing the type of reasoning used by those students, Krutetskii (1976) identified three main categories: analytic (non- visual), geometric (visual) and harmonic (use of the two previous types of reasoning). In spite of the existence of different approaches to the same problem, most students prefer to use numerical relations as a support for reasoning, perhaps reflecting the work promoted in the classroom where analytic representations prevail. However, some studies indicate that most students are more successful when they use a harmonic or mixed approach (Noss, Healy & Hoyles, 1997; Stacey, 1989; Becker & Rivera, 2005).

The relation between the use of visual abilities and students' mathematical performance constitutes an interesting area for research. Many researchers stress the importance of the role visualization plays in problem solving (Presmeg, 2006; Shama & Dreyfus, 1994), while others claim that visualization should only be used as a complement to analytic reasoning (Goldenberg, 1996; Tall, 1991). In spite of some controversy, these visions reflect the importance of using and developing visual abilities in mathematics but teachers tend to present visual reasoning only as a possible strategy for problem solving in an initial stage or, when necessary, as a complement to analytic methods (Presmeg, 1986). Several studies point to the potential of visual approaches for supporting problem solving and mathematical learning. The reality of our classrooms, however, tells us that students display frequently reluctance to exploit visual support systems (Dreyfus, 1991) and tend not to make links between visual and analytical thought (Presmeg, 1986). These ideas imply that the role of visualization in school mathematics should be re-evaluated.

Students' thinking processes in pattern generalization

There are some studies exploring students' difficulties and strategies, from pre-kindergarten to secondary school, when solving problems requiring pattern seeking. Their results are discussed in this section.

Stacey (1989) focused her research on the generalization of linear patterns, by students aged 9-13 years old. She classified students' strategies when solving contextualized linear generalization tasks, whether or not leading to correct answers. Strategies found were: *counting*, *whole-object*, *difference* and *linear*. In the *counting* strategy, students counted the number of items in a figure. Those who employed the *whole-object* strategy used a multiple of a previous value, assuming the problem implied direct variation. The *difference* strategy consisted of using a multiple of the difference between two consecutive items of the sequence. Finally, students who used the *linear* strategy applied a linear model to find solutions. In her study, Stacey

(1989) concluded that a significant number of students used an incorrect direct proportion method when attempting to generalize.

García Cruz & Martinón (1997) developed a study aiming to analyse the processes of generalization developed by secondary school students. Their categorization of the methods used by these students was based on Stacey's work. They considered three main categories: *counting* (included counting the items on a drawing and extending a sequence using a recursive method), *direct proportion* and *linear*. They have also classified strategies according to their nature: *visual*, *numeric* and *mixed*. If the drawing played an essential role in finding the pattern it was considered a *visual* strategy, on the other hand, if the basis for finding the pattern was the numeric sequence then the strategy was considered *numeric*. Students who used *mixed* strategies acted mainly on the numeric sequence and used the drawing as a means to verify the validity of the solution. Results of this research have shown that the drawing played a double role in the process of abstracting and generalizing. It represented the setting for students who used visual strategies in order to achieve generalization and acted as a means to check the validity of the reasoning for students who favoured numeric strategies.

Orton and Orton (1999) focused their research on linear and quadratic patterns with 10-13 years old students. They reported a tendency to use differences between consecutive elements, as a strategy in the generalization of linear patterns, and its extension to quadratic patterns, by taking second differences, but without success in some cases. They also pointed as obstacles to successful generalization, students' arithmetical incompetence and their fixation on a recursive approach that, although being useful in solving near generalization tasks, doesn't contribute to the understanding of the structure of a pattern.

In a more recent study, Becker and Rivera (2005) described 9th grade students work after they were asked to perform generalizations on a task involving linear patterns. They tried to analyse successful strategies students used to develop an explicit generalization and to understand their use of visual and numerical cues. The researchers found that students' strategies appeared to be predominantly numeric and identified three types of generalization: *numerical*, *figural* and *pragmatic*. Students using numerical generalization employed trial and error with little sense of what the coefficients in the linear pattern represented. Those who used *figural* generalization focused on relations between numbers in the sequence and were capable of seeing variables within the context of a functional relationship. Students who used *pragmatic* generalization employed both numerical and figural strategies, seeing sequences of numbers as consisting of both properties and relationships.

Method

Fifty four sixth-grade students (11-12 years old), from three different schools in the North of Portugal, participated in this study over the course of a school year. The study was divided in three stages: the first corresponded to the administration of a test focusing on pattern exploration and generalization problems; second stage, which went on for nearly 6 months, involved all students in each classroom solving patterning tasks, in pairs; and, on the third, students repeated the test in order for us to examine changes in the results. These students were described by their teachers as being of average ability and had no prior experience with this kind of tasks. Over the school year all students involved in the study solved seven tasks and two pairs from each school were selected for clinical interviews. Students' activity when solving the tasks was videotaped and transcribed for further analysis.

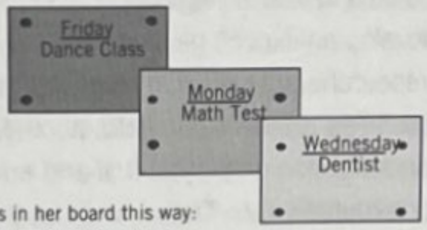
Analysis of the Pins and cards task

This is an ongoing study and at present time some of the data is still being analyzed. In this paper we will focus on some preliminary results based on the application of one of the tasks.

Along the study we applied a total of seven tasks. They involved near and far generalization and featured increasing and decreasing linear patterns as well as non linear ones. In the selection process we tried to privilege tasks whose nature could lead to the use of different strategies, allowing students to find patterns in either number or visual solving contexts. One of the tasks used in this study was called *Pins and Cards* (see figure 1) and involved the generalization of an increasing linear pattern.

Pins and Cards

Joana hangs cards on a board in her room in order to remember her appointments. She uses pins to support the cards as shown in the image.



If she continues to hang cards in her board this way:

1. How many pins will she need to hang 6 cards?
2. What if she was to hang 35 cards, how many pins would she need?
3. Supposing that Joana bought a box with 600 pins, how many cards can she hang in her board?
4. Joana decided to use triangular cards. Knowing that she sticks a pin on each vertice of the triangle and consecutive triangles have one pin in common, anal yze the previous questions in this context.

Figure 1. *Pins and cards task*

The analysis of the work developed in this task allowed us to identify a diversity of strategies, as well as difficulties, that students had also shown on the pre-test. We felt the need to adjust Stacey's (1989) strategy categorization in order to describe, as accurate as possible, students' reasoning, establishing four main categories: ***counting***, ***whole-object***, ***recursive*** and *linear*. In some cases we considered that a particular category had to be divided in different approaches due to the structure of reasoning presented.

Most of the tasks we designed had a strong visual component, as we can see in the *Pins and Cards* task. Near generalization questions (1. and 4.1) can easily be solved by making a drawing of the requested term of the sequence and counting its elements, using what Stacey (1989) called the *counting* strategy (*C*).

The *whole-object* strategy (Stacey, 1989) also emerged from the work of some of the pairs. This approach is associated to direct proportion situations and this particular problem, as others presented along the study, does not fit this model. For this strategy to be adequate, students had to make a final adjustment based on the context. We identified two different ways in which students applied the *whole-object* strategy (*W*), without any adjustment, leading to incorrect answers: (*W*1) using multiples of a given term of the sequence; (*W*2) using multiples of different terms of the sequence and adding them (in this case the requested term is obtained by decomposition, using known elements of the sequence).

This type of tasks can promote the use of *recursive* thinking, especially when near generalization is involved. So it came as no surprise that some students used the common difference between two consecutive terms of the sequence to solve some of the questions posed. We distinguished two situations in which this strategy was employed: (*R*1) extending the sequence using the common difference; (*R*2) using multiples of the common difference (as happened with the *whole-object* strategy, for this reasoning to be adequate students needed to adjust the result based on the context of the problem. Thus, in this case, the answer was incorrect).

The *linear* strategy (Stacey, 1989) relates to the use of expressions of the type $an+b$ ($b \neq 0$). In this study we identified four categories that are in some way linked to this particular strategy: (*L*1) identifying an explicit rule that relates the order of a given term of the sequence with the number of elements of that term (in this particular case students "saw" that each card needed three pins and the last one would need four, deducing that the rule was $3(n-1)+4$, n being the number of cards. Other pairs "saw" the pattern differently considering that each card had three pins adding one more pin at the end. Here the rule was $3n+1$); (*L*2) using multiples of a given term of the sequence and making a final adjustment based on the context of the problem; (*L*3) using multiples of a given term of the sequence and making a final adjustment based only on numeric relations (this approach is based on the *whole-object* strategy and the adjustment made is disconnected from the context leading to an incorrect answer); (*L*4) applying a model based on recursive thinking (in this case, students used multiples of the common difference and made a final adjustment based on the context of the problem).

In table 1 we present the number of answers in each of the categories described above concerning the *Pins and Cards* task. In some cases we couldn't categorize students' answers so those cases appear in the last column, not categorized (*NC*). This table allowed us to analyze not only the method used to solve each question but also the context in which it was applied.

Table 1. *Summary of students' responses*

	C	W1	W2	W	R1	R2	R	L1	L2	L3	L4	L	NC
1.	16	5	-	5	1	1	2	1	-	-	-	1	3
2.	-	-	-	-	3	1	4	13	1	2	-	16	7
3.	-	2	1	3	-	4	4	10	1	1	3	15	5
4.1	17	3	-	3	-	1	1	-	-	-	-	-	6
4.2	1	-	-	-	1	1	2	11	-	1	-	12	12
4.3	-	1	-	1	-	4	4	8	-	-	3	11	11

As we can see from Table 1, when solving near generalization questions (1 and 4.1) the predominant strategy was *counting* over a drawing. The other strategies were also applied but only a minority of pairs used them, in spite of being highlighted in the existing literature that students focus on recurrence relationships when dealing with patterns (Stacey, 1989; Orton & Orton, 1999). In far generalization questions (2, 3, 4.2 and 4.3) the application of *linear* strategies prevailed.

When solving this task some students struggled with cognitive difficulties that led to incorrect answers. Some pairs made false assumptions about the use of direct proportion. In these cases attention tended to be focused only on numeric attributes with no appreciation of the structure of the sequence. This happened with strategies *W1*, *W2* and *L3*, where the only concern was to satisfy numeric relations. The use of strategies based on recursive reasoning wasn't always made correctly, especially when far generalization questions were involved. The recursive approach through the use of *R2* lacked a final adjustment based on the context of the problem, because students only considered a multiple of the common difference, forgetting to add the last four pins or the last pin. Also, when they used linear strategies, the model wasn't always correctly applied. We have the example mentioned above with the use of *L3* and, in some cases, strategy *L1* wasn't totally correct as students added pins and cards in the end. We are convinced that these errors are linked to the extensive experience of students in manipulating numbers without meaning, making no sense of what the coefficients in the linear pattern represent.

According to Presmeg (1986) a strategy is considered visual if the image/drawing plays a central role in obtaining the answer, either directly or as a starting point for finding the rule. In this sense we believe that *counting* and *linear* strategies *L1*, *L2* and *L4* are included in this group. *Counting* was always a successful strategy but only useful in solving near generalization questions. Drawing a picture of the object required and counting all the elements is an action used in near generalization questions and does not lead to a generalized strategy. The *linear* strategy *L1* lead to a correct answer when students based their work on the structure of the sequence, making reference to the way pins were distributed through the cards. Strategy *L2* was only used by one student but did not led to a correct answer. We think that it involved a higher level of abstraction in visualization, difficult to attain in this stage of the study. Finally, only three pairs of students used *linear* strategy *L4*. All of these students reached the correct answer. They considered that each card had three pins and made a final adjustment considering that the last card would need one more pin.

Final discussion

The main purpose of this study was to characterize students thinking processes when exploring pattern problems, focusing on their generalization strategies and most common difficulties. Results are based on the performance of fifty-four 6th grade students who hadn't yet had any formal algebra instruction. They solved a linear pattern task, presented with a visual support, with the aim to facilitate the apprehension of the structure of the sequence. The task was scaffolded in a way that allowed students to achieve generalization gradually, so the questions proposed were arranged in a sequence that required first near generalization, then far generalization (Stacey, 1989).

As for the research questions outlined earlier in this paper, we can now make some remarks:

1. Students presented several difficulties in solving problems involving pattern exploration, especially when they had to generalize for a distant term. They've achieved better results in questions that involved near generalization than on far generalization. The structure of the pattern on this task may have induced some of the difficulties presented by students. They had limited experiences with visual linear increasing patterns in previous years and, on the other hand, a presumable inability to visualize spatially (Warren, 2000) may have constituted a key obstacle to an adequate exploration.
2. A variety of strategies has been identified in the work developed by students, converging with what has been written in the literature (Becker & Rivera, 2005; García Cruz & Martinón, 1997; Orton & Orton, 1999; Stacey, 1989). However some of the strategies applied by these students were more frequent than others, depending on the nature of the question involved. Orton et al. (1999) and Stacey and MacGregor (2001) found that students often choose to reason recursively when exploring patterning tasks. In this study, despite being one of the strategies applied, recursive reasoning was not the predominant approach. Students usually used the *counting* strategy to solve near generalization questions, drawing representations of the terms of the sequence and summing up their elements. In far generalization, questions we noted a shift on the predominant strategy towards a *linear* type.
3. The *Principles and Standards for School Mathematics* (NCTM, 2000) strongly recommends integrating visualization strategies in students' mathematical experiences throughout content areas. But this is not an easy task. Duval (2006) points out that there are many ways of "seeing". Students can see a figure in an iconic way or in a mathematical way. In this sense the act of observing and exploring patterns, presented in a visual form, may follow certain conventional practices. For instance, many students have developed a misconception that mathematics is reduced to the manipulation of numbers and numerical expressions and, in this case, visualizing comes as an unnecessary act for them. In this study, some of the pairs that worked exclusively on number contexts used inadequate strategies like: the application of direct proportion; using multiples of

the difference between two consecutive terms without a final adjustment; and mixing variables, in this case, pins and cards. On the other hand, visualization proved to be a useful ability in different situations like making a drawing and counting its elements to reach near generalization. Also for "seeing" the structure of the pattern that allowed to discover a linear strategy, achieving far generalization. So we think that it's important to provide tasks which encourage students to: a) use and understand the potential of visual strategies; b) relate number context with visual context to later understand number sense and variable meaning.

The process of generalizing based on visual cues often contributes to an insightful interpretation of the structure of the pattern and simultaneously to the proposal of multiple expressions of the same pattern, depending on the way it is seen. This situation is not likely to happen when only numerical approaches are applied. The possibility of having multiple expressions of generality for the same pattern is of crucial importance in the classroom, leading to the discussion of several concepts, particularly of equivalent expressions. Hence, visualization in algebra constitutes an alternative way to understand structures and relations between variables.

In our opinion, future research should continue to focus on analyzing and developing generalizing strategies, of visual and numerical nature. We also believe that this possibility of applying multiple approaches in the generalization of the same pattern, should lead teachers to encourage their students to share strategies and to interpret them, deepening their repertoire.

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