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A Math Education Laboratory: a space for active learning and pedagogical innovation

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Abstract: School mathematics requires effective teaching that engages students in meaningful learning experiences, both individually and collaboratively, providing them with opportunities to communicate, reason, be creative, think critically, solve problems, and make decisions. In this approach, students are active individuals involved in their learning process, shaped by intellectual, social and physical interactions. So, future teachers should implement practices that lead their own students to develop these skills, which underscores the importance of teacher training programs designed to meet these challenges. This paper presents an exploratory qualitative study aimed at characterizing the Math Education Lab at our institution, the LEM, focusing on its role in teacher training programs to enhance pedagogical practices and improve students' mathematical knowledge. In particular, we are interested in: characterizing the evolution of the LEM since its creation to the present day; describing activities experienced by preservice teachers in the LEM; and identifying future teachers' perceptions about the LEM's role in their professional development. Data were collected through a narrative design, based on documents, a questionnaire, and the personal experiences of two trainers/researchers who have engaged with the LEM over its existence. Preliminary results show that the LEM has evolved from a mere repository of manipulative materials, into a dynamic space where active learning of mathematics is worked on and research is carried out. Future teachers express the LEM's importance as a privileged space to work on mathematics in different ways, in accordance with current trends, useful for their personal learning and actualization, but also as future teachers.
Keywords: Math education lab. Preservice teachers. Active learning. Authentic and hands-on tasks.

Laboratório de Educação Matemática: um espaço de aprendizagem ativa e inovação pedagógica

Resumo: A matemática escolar requer um ensino eficaz que envolva os alunos em experiências de aprendizagem significativas, individuais e colaborativas, dando-lhes oportunidades para comunicar, raciocinar, ser criativos, pensar criticamente, resolver problemas e tomar decisões. Neste contexto, os alunos são indivíduos ativos, envolvidos na sua aprendizagem, resultante da interação entre as dimensões intelectual, social e física. Assim, os futuros professores devem implementar práticas que levem os alunos a desenvolver essas capacidades, sendo importante ter programas de formação projetados para enfrentar esses desafios.

Apresentamos um estudo qualitativo exploratório que teve como objetivo caraterizar o Laboratório de Educação Matemática da nossa instituição, o LEM, com foco no seu papel na formação de professores perspetivando a melhoria das práticas pedagógicas e o desenvolvimento do conhecimento matemático desses futuros professores. Em particular, pretendemos: caraterizar a evolução do LEM desde a sua criação; descrever atividades experienciadas pelos futuros professores no LEM; e identificar as suas perceções destes futuros professores sobre o papel do LEM no seu desenvolvimento profissional. Os dados foram recolhidos através de uma abordagem narrativa, baseada em documentos, num questionário e nas experiências pessoais de duas formadoras ligadas ao LEM desde a sua criação. Resultados preliminares mostram que o LEM evoluiu de um repositório de materiais manipuláveis, para um espaço dinâmico onde se trabalha uma matemática ativa e se realiza investigação. Estes estudantes expressam a importância do LEM como um espaço privilegiado para trabalhar a matemática de diferentes formas, de acordo com as tendências atuais, sendo útil para a sua aprendizagem e atualização, como para a sua atuação como futuros professores.
Palavras-chave: Laboratório de educação matemática. Professores em formação. Aprendizagem ativa. Tarefas autênticas e hands-on.

Laboratorio de Educación Matemática: u espacio de aprendizaje activo y innovación pedagógica

Resumen: La matemática escolar requiere una enseñanza efectiva que involucre a los alumnos en experiencias de aprendizaje significativas, individuales e colaborativas, proporcionándoles oportunidades para comunicarse, razonar, ser creativos, pensar críticamente, resolver problemas y tomar decisiones. Ádemas, los alumnos son individuos activos implicados en su aprendizaje, aprendizaje, moldeada por interacciones intelectuales, sociales y físicas. Así, los futuros profesores deben aplicar prácticas que lleven a sus propios alumnos a desarrollar estas habilidades esenciales, siendo los programas de formación diseñados para afrontar estos desafíos. Presentamos un estudio cualitativo exploratorio com objetivo caracterizar el Laboratorio de Educación Matemática de nuestra institución, LEM, en los programas de formación para mejorar las prácticas pedagógicas y mejorar el conocimiento matemático de los estudiantes. En concreto deseamos: caracterizar la evolución del LEM desde su creación; describir actividades experimentales por los profesores en el LEM; e identificar sus percepciones sobre el papel del LEM en su desarrollo profesional. Los datos se recogieron mediante un diseño narrativo, basado en documentos, un cuestionario y las experiencias personales dos formadores que han participado en el LEM. Los resultados preliminares muestran que el LEM ha pasado de un depósito de materiales manipulativos a un espacio dinámico donde se trabaja el aprendizaje activo de las matemáticas y se investiga. Los futuros profesores manifiestan la importancia del LEM como espacio privilegiado para trabajar las matemáticas de diferentes maneras, de acuerdo con las tendencias actuales, útiles para su aprendizaje y actualización personal, pero también como futuros profesores de matemáticas.
Palavras-Clave: Laboratorio de educación matemática. Profesores en formación. Aprendizaje activo. Tareas auténticas e hands-on.

Introduction

Social and economic changes over recent decades have significantly affected society, the job market, and consequently, education, particularly in the 21st century. Schools now face constant and increasingly complex challenges to motivate students to learn in a world where information is just a click away. This reality requires that students develop a set of higher-order skills essential for navigating these complexities as problem solving and critical thinking, communication and collaboration, which enable effective team work in diverse settings and creativity connected to innovation, encouraged through open-ended tasks and projects (OECD, 2019).

To foster meaningful learning, it is essential to create a classroom environment where students feel both challenged and motivated to learn, applying an exploratory teaching (Ponte, 2014; Vale; Barbosa, 2023b) that promotes active learning (Edwards, 2015; Prince, 2004), through the use of rich, challenging tasks with multiple solutions (Leikin, 2016) that stimulate the development of the abovementioned skills and encourage student involvement, increasing their understanding (Jurdak, 2006). We prioritized authentic tasks (NCTM, 2014; OECD, 2019) that simulate real-world problem solving (Barbosa *et al.*, 2024; Vale, 2024; Jurdak, 2006; Vale *et al.*, 2023), with multiple connections (within and outside mathematics) between concepts, realities and disciplines (Barbosa *et al.,* 2024). Furthermore, the practices developed must meet the diversity of students' learning styles, serving as a stimulus to the development of creativity (Borromeo-Ferri, 2013).

In this context, the creation of spaces like a Mathematics Education Laboratory or Math Laboratory (Math Lab), aim to respond to an urgent need to reformulate the way mathematics is taught and learned, especially in the context of teacher training (Anaduaka; Sunday, 2021; Lorenzato, 2006; Silva, 2020). They emerge as an environment with a focus on 21st-century skills, that allows the incorporation of active learning through hands-on and minds-on tasks, creating a dynamic educational setting that prepares students for future challenges. By engaging students in meaningful ways, educators can foster a deeper understanding of content and develop essential skills for lifelong learning (Edwards, 2015; Vale *et al.*, 2023). The quality of the students' learning is strongly dependent on the teachers' choices, particularly regarding the nature of the tasks proposed and the strategies used to challenge and engage students (Ponte, 2014; Vale; Barbosa, 2023b).

In the evolving landscape of education, the integration of hands-on learning experiences has become increasingly vital, particularly in the field of mathematics. Math Labs provide interactive and practical environments for students to explore mathematical concepts, establishing themselves as a powerful space in math education. These labs facilitate a deeper understanding of mathematical principles by allowing students to engage with the material in a tangible way, fostering several skills. The integration of Mathematics Education Laboratories into teacher training programs has emerged as a pivotal area of research and practice. The theoretical framework that supports the activities occurring in these labs draws on the intersection of teaching and learning principles, constructivist approaches, enhancing teacher efficacy and student outcomes. Research has shown that experiential learning in math labs can significantly improve student engagement and comprehension (Clements, Sarama, 2009; NCTM, 2014). Also, by incorporating technology and collaborative activities, math labs create an inclusive atmosphere that accommodate a diversity of learning styles, ultimately leading to improved academic outcomes (Hattie, 2009).

In accordance with the previous ideas, our purpose is to describe and reflect about what a Math Lab is and the work done in this space. This reflection seeks to identify key components that influence and contribute to the effectiveness of such a laboratory in the teaching and learning process, as well as in teacher education. Specifically, we aim to understand how a Math Lab can improve teacher training and, ultimately, improve students' mathematics learning outcomes. Thus, our objective is to explore the purpose and impact of a Math Lab, identifying the elements that make it an effective space for fostering improvement in mathematics education at both the teacher and student levels. The following section, theoretical framework, outlines the perspectives of key components underlying such laboratories.

Teaching and learning of mathematics

Teacher training practices directly influence teacher quality and, indirectly, student learning, which leads teacher trainers to take on a central role in developing and leading these processes, facilitating learning environments that support the professional development of these (future) teachers. So, the main responsibility of teachers is to create rich learning opportunities for their students to acquire and demonstrate not only what they know but also what they are able to do with that knowledge.

Teaching and learning mathematics in contemporary educational contexts demand innovative approaches that actively and holistically engage students. Active learning has emerged as a pillar of effective mathematics education, emphasizing the importance of intellectual, social, and physical engagement in the learning process. Grounded in socio-constructivist theories, particularly Vygotsky's emphasis on social interaction in knowledge construction (Vygotsky, 1996), active learning fosters deeper understanding by encouraging students to interact with tasks, peers, and the environment. This approach contrasts with traditional, passive learning models, where knowledge is transmitted by the teacher and memorized by the students (NCTM, 2014). Instead, active learning places students at the center of the learning process, promoting exploration, collaboration, and creativity (Prince, 2004; Vale; Barbosa, 2023a). This demands for an exploratory teaching, where the nature of the tasks and discussions among students and teacher are crucial, provoking reflection and where the different approaches and perspectives are valued (Ponte, 2014; Vale; Barbosa, 2023b). Active learning strategies, such as problem-solving, math trails, and gallery walks illustrate this transformative approach. These methods allow learners to explore concepts, reflect on their learning, and collaboratively build mathematical knowledge (Vale; Barbosa, 2023a). Research shows that experiential learning activities in classrooms not only improve students' engagement but also enhance their problem-solving abilities and critical thinking skills (Clements; Sarama, 2009; Llinares, 2021). Organizations like the National Council of Teachers of Mathematics (NCTM, 2014) and the OECD (2019) strongly advocate for active learning as a means to prepare students for the complexities of the modern world. The Ontario Ministry of Education (OME, 2016) also highlights the integration of the 4Cs of 21st-century education - Critical Thinking, Communication, Collaboration, and Creativity - as essential for equipping learners with the skills necessary to deal with an interconnected and rapidly evolving global society. Active learning in mathematics directly supports these competencies. Critical thinking is promoted through tasks that require analysis, evaluation, and the development of innovative solutions (Leikin, 2016). Problem-solving tasks with multiple solutions challenge students to think flexibly and adaptively (Pólya, 1978). Communication is emphasized in activities where students articulate their reasoning, share ideas, and discuss their strategies with peers, such as a gallery walk (Vale; Barbosa, 2021). Collaboration naturally arises in group-based tasks and discussions, fostering a sense of community in tackling mathematical problems. Finally, creativity is encouraged through tasks that allow diverse approaches, multiple solutions and innovative thinking, helping students see mathematics not merely as a set of rules but as a dynamic field of exploration (Leikin, 2016).

Effective mathematics teaching should also recognize the diversity of learning styles among students. Drawing on Krutetskii's (1976) classification, learners can be broadly categorized as visual, verbal-logical, and harmonic thinkers. This diversity underscores the importance of differentiated instruction and the need for multiple approaches. Visual learners benefit from diagrams, graphs, and spatial tasks that make abstract concepts tangible (Borromeo-Ferri, 2013).

Verbal-logical learners thrive on algebraic and numerical representations, while harmonic learners, who integrate both styles, require tasks that balance visual and logical elements. By incorporating varied instructional approaches, teachers can address the unique preferences of each learner, ensuring inclusivity and fostering deeper engagement (Borromeo-Ferri, 2012). The teacher's role in creating and facilitating active learning environments is pivotal. Effective task design is essential, emphasizing inquiry-based learning and productive struggle. Tasks should challenge students to explore concepts, draw connections, and think critically while maintaining a balance that prevents frustration (NCTM, 2014). Multiple-solution tasks are particularly valuable, as they not only accommodate diverse thinking styles but also foster creativity and flexibility. Through these tasks, students gain confidence in their problem-solving abilities and develop a deeper appreciation for the richness of mathematics (Leikin, 2016).

Finally, teaching and learning mathematics through active, collaborative, and distinguished approaches align with the demands of contemporary education, can transform the mathematics classroom into a space of exploration, discovery, and growth. Supported by international research and frameworks (Leikin, 2016; Llinares, 2021; NCTM, 2014; OECD, 2019), this approach ensures that students are not only equipped with mathematical knowledge but also with the critical skills and mindsets needed to navigate an increasingly complex and interconnected world.

The Math Laboratories

Mathematics Education Laboratories represent innovative and transformative spaces that play a central role in both the teaching and learning of mathematics and in the education of future teachers. These laboratories provide a dynamic environment where students and educators can explore, reflect, and collaborate to deepen their understanding of mathematical concepts.

The creation of these spaces responds to an urgent need to reformulate the way mathematics is taught and learned, especially in the context of teacher training (Lorenzato, 2006; Silva *et al.*, 2024; Silva, 2020), emphasizing that these laboratories bridge the gap between abstract theory and practical application, making mathematics more meaningful and accessible. Students engage in hands-on activities that allow them to verify mathematical properties, explore relationships, and connect mathematics to real-world scenarios, fostering curiosity and logical reasoning. Di Sia (2014) highlights the dual role of Math Labs as physical spaces but also as creators of moments of active engagement, where students discuss, experiment, and build connections between intuitive and formal mathematics. Such spaces are not merely repositories of materials but are structured to facilitate reflective practice and critical inquiry, enabling learners to transition from basic operational levels to advanced mathematical thinking (Meurer *et al.*, 2023). Sharing these ideas, but specifying, Giberti (2022) says that a Math Lab goes further than just a classroom, it can be described as a structured framework of activities designed to foster the construction of meaning around mathematical concepts. This space integrates various components: people (students and teachers), structures (classrooms, tools, spatial and temporal organization), and ideas (teaching plans, projects, and experiments). The environment allows the learning was achieved through hands-on experiences, observation, collaboration, and dialogue with both peers and experts.

In the context of teacher education, mathematics laboratories serve as critical spaces for professional development and pedagogical innovation. They allow pre-service and in-service teachers to engage with innovative methodologies and resources. In this context, Shulman's (1986) concept of pedagogical content knowledge becomes central, as teachers integrate content mastery with effective instructional strategies. According to Silva *et al.* (2024), these environments enable pre-service teachers to connect theoretical knowledge with practical teaching strategies. They allow educators to experiment with diverse methodologies, reflect on their instructional practices, and develop a deeper understanding of how to create engaging and effective learning experiences. Active learning lies at the heart of Math Labs, as they emphasize intellectual, social, and physical engagement in the learning process (Edwards, 2015). The socio-constructivist framework underpinning these spaces, inspired by Vygotsky's (1996) theories, promotes meaningful interactions among students, peers, and their environments. Tasks such as problem-solving, conjecturing, and collaborative discussions are common, enabling students to develop critical thinking and problem-solving skills. The 4Cs are also naturally aligned with the activities in these spaces.

The function of Math Labs extends beyond individual learning to support broader educational goals. Meurer *et al.* (2023) describe various types of laboratories, including those designed specifically as "training agents" for teacher education. These laboratories foster collaboration, reflective practices, and the development of innovative teaching strategies. They serve as hubs where educators can share ideas, design lessons, and develop materials that align with contemporary educational standards. In addition to supporting teacher education, mathematics laboratories enhance the curriculum by aligning hands-on activities with educational objectives. Anaduaka and Sunday (2021) argue that these activities allow students to see the relevance of mathematics in real-world contexts, making learning more meaningful and engaging. By connecting mathematical concepts with interdisciplinary applications in fields like physics, economics, and engineering, laboratories help students develop a comprehensive understanding of how mathematics influences everyday life and future career paths. Despite their transformative potential, the implementation of mathematics laboratories is not without challenges. Silva *et al.* (2024) and Oliveira and Kikuchi (2018) highlight issues such as limited resources, insufficient teacher training, and the need for greater institutional support. Overcoming these obstacles requires targeted investments in materials, infrastructure, and professional development. With adequate support, mathematics laboratories can fulfill their potential as spaces that inspire both students and educators.

Finally, mathematics laboratories are essential for reimagining the teaching and learning of mathematics. They provide opportunities for students to engage actively with mathematical concepts and for teachers to refine their pedagogical practices. By fostering collaboration, creativity, and critical thinking, these spaces address the needs of 21st-century education and prepare learners and educators to meet future challenges. With a commitment to address implementation challenges, mathematics laboratories can become central to transform mathematics education.

Methodology and procedures

In this paper we followed an exploratory qualitative research approach (Clandinin; Connelly, 2000; Creswell, 2009) to understand the role and impact of the LEM, identifying key components that influence its effectiveness in the teaching and learning process, as well as in teacher education. To better address this purpose, we decided to focus on specific ideas that were translated into the following research questions: (1) How can we characterize the evolution of the LEM since its creation to the present day?; (2) Which potentials and challenges can be identified in the activities developed/experienced within the LEM?; and (3) How do future teachers perceive the role and potentialities of the LEM in their professional development? This study adopts a qualitative approach, reflecting the subjectivity inherent to the research problem (Creswell, 2009). Data were collected in a holistic, descriptive and interpretative way (Creswell, 2009; Miles; Huberman, 1994). This process was mainly grounded in a narrative design, based on the different perspectives drawn from the vivid personal experiences of the LEM coordinator over the years, as well as insights from researchers and users (students). Multiple sources of evidence were utilized, including documents (written documents, record notes, written productions), photographs, artifacts, observations, and a questionnaire applied to the students (Clandinin; Connelly, 2000; Creswell, 2009).

This study was deeply immersed in the experiences and thoughts of all the participants, including two researchers and 10 students, future teachers. The researchers are educators specializing in didactics of mathematics within teacher training courses, with one serving as the coordinator of the LEM since its creation. The student participants were 10 preservice teachers enrolled in the first semester of the third year of a Master's program in Primary Education, with specialization in Mathematics and Sciences (future teachers of children aged 6–12 years). This course was composed of subjects related to the areas of didactics, general education, content knowledge, and practice in educational contexts. All didactics classes took place in the LEM space, conducted by the researchers. This setting provided a dynamic environment for active learning experiences and also served as the central context for data collection.

In this space, participants experienced a variety of tasks. For this manuscript, we selected two specific tasks to illustrate the work proposed by the researchers and undertaken by the students. Additionally, an exploratory questionnaire was applied to the students to identify the main thoughts and perceptions about the LEM's role in their professional development. This open questionnaire is an adaptation from those of two authors (Dittmer, 1971; Silva, 2020). It consists of two parts: the first focuses on the LEM as a physical space (e.g., How would you define the LEM?; What are the main differences between the LEM and other classrooms?; What resources should a LEM include?; Do you consider the LEM a repository of material resources?); the second part focuses on the LEM as a space for teaching and learning (e.g., Do you feel more motivated in mathematics classes conducted in this space? What do you consider to be the greatest contributions the LEM can offer you as a student and as a future teacher?). We consider that future teachers' perceptions are shaped by a combination of personal experiences, organized training, and reflective practices, all of which play a crucial role in shaping their teaching identity and effectiveness in the classroom, contributing for their professional development (Feiman-Nemser, 2001; Shulman, 1986). The analysis of the questionnaire is framed in the scope of its questions through qualitative content analysis and the lenses of the researchers (Miles; Huberman, 1994). In addition to the questionnaire, we also drew upon the researchers' memories, experiences, reflections and documents, detailing the

LEM's creation and evolution. The dual role of the researchers as both teachers and responsible for the unit course provided a unique opportunity for naturalistic participant observation. This enabled the recording of free-flowing notes that captured the preservice teachers' reactions, interactions, conversations, discussions, and interpretations. These notes included a record of facts but also the researchers' commentaries about those facts.

Grounded on the research questions, the data collected and the literature review carried out, we organized the data into three following sections: A short snapshot of the LEM; LEM - a space for teaching, learning and research; and Some students' perceptions about the LEM.

A short snapshot of the LEM

The LEM holds the distinction of being the first of its kind in Portugal. In this section, we provide a brief overview of the LEM creation, objectives, and the activities developed throughout its existence. For more detailed information of the LEM, see the paper by Silva *et al.* (2024).

For over 30 years, the LEM has been a privileged space of one researcher's professional journey as mathematics educator, serving not only her own training but also in the professional development of future teachers at ESEVC. It has functioned as both an incubator for addressing various problems under study, both in the scope of didactics of mathematics and in teacher training in the early years. The primary aim of the LEM is to support teachers rethinking their didactical practices, in order to promote engaging and relevant activities through which we explore the different types of knowledge that future teachers need to develop, in order to provide the foundations of their professional development.

The LEM - space and mission

The LEM was created in 1989 under the leadership of Professor Domingos Fernandes, the first mathematics education professor at the School of Education of the Polytechnic Institute of Viana do Castelo (ESEVC), along with other colleagues from the institution, including the first author of this paper. Initially, the ESEVC focused on training pre-service teachers for the elementary education (ages 3-12 years) and providing in-service teacher training for all levels. Over the years, the LEM has undergone significant transformations in terms of physical space, equipment and extended some of its objectives. Since its creation, the mission of the LEM (Vale *et al.*, 2022) has been centered around three main areas: training, research, and community outreach. In particular, in this document we highlight that "The LEM aims to produce, develop and disseminate scientific, pedagogical and didactic knowledge in the area of mathematics education, carrying out research in context, and providing services to the community" (p.1).

Figure 1 shows the evolution of the LEM's spaces. The first two images depict the initial spaces, while the subsequent two show the LEM during its preparation phase. The last image presents the fully operational, actual LEM. As we can see, currently the space of the LEM consists of two independent rooms, that can be separated through a movable door, allowing a flexible use. Between these two rooms is a long central table, facilitating collaborative work and group activities.

Figure 1. The LEM spaces

Source: Authors

The use of the LEM, as either a single unified room or two separate spaces, depends on the type of activities being developed. The LEM is currently well-equipped with a wide range of bibliographic materials (e.g., books, textbooks, booklets, magazines - mainly productions of the *NCTM*-National Council of Teachers of Mathematics; and of the *APM*-Associação de Professores de Matemática [Association of Mathematics Teachers of Portugal), manipulatives (e.g., geoboards, tangrams, pattern blocks, cuisenaire and Dienes materials), and technological devices (e.g., computers, tablets, 3D printer, interactive touch board, wireless system). Notably, the LEM is equipped with a remote communication system that enables activities ranging from conferences to virtual classes. This system facilitates real-time visualization of the LEM space and the work being carried out by participants, fostering remote communication and collaboration. This resource proved especially valuable during the COVID-19 pandemic. To address instances where the physical space of LEM cannot accommodate all the demands of the different disciplines and courses - especially those requiring manipulative materials - two portable trolleys are available. These trolleys allow the convenient transport of necessary materials, ensuring the seamless continuation of classes and activities outside the LEM. We consider the LEM to be well-equipped; however, our most pressing need is to acquire more versatile furniture that allows for quick reconfiguration of the space to suit the nature of the planned activities and the needs of its users.

When the LEM was first established (in the 1990s), one of its primary objectives was to support preservice teachers in their pedagogical practice across the different courses offered by ESEVC, particularly in the field of mathematics education. Additionally, the LEM served as resource for local teachers in general, by making teaching materials available to schools in the district. Over time, as the needs and challenges of mathematics education evolved, the objectives of the LEM also had to adapt. Although we continue to support students and schools by providing teaching materials for their classes or exhibitions, the scope of the LEM has broadened significantly: “a) Designing, developing, supporting and evaluating teaching and learning trajectories and strategies, as well as creative and innovative teaching materials, based on the latest research trends in mathematics education at different levels of education (Vale *et al.*, 2022, p. 2)”

In light of the actual trends of the society and the challenges that education faces, we continue to contribute to improve the (initial) training of teachers, preparing them with diversified and current educational strategies and tools.

LEM- an *atelier* for the construction of didactical materials

Since its creation, one of the core functions of the LEM has been the construction of educational materials; encompassing all stages from conception to construction, editing, and publication. Another objective of the LEM, as stated by Vale *et al.* (2022, p. 3), is: “*h) Design, create and publish documents and materials to support research and pedagogical action”*. The commitment to producing high-quality resources continues today. The LEM regularly develops and provides materials for both students and teachers, including a variety of games and puzzles (Figure 2).

Figure 2: Some LEM games/puzzles

Source: Authors

Initially, many materials necessary for the teaching and learning process were either not available on the market or quite expensive. As a result, some were created by the teachers, while others were built by students under the supervision of their teachers. Among these, we would like to highlight three manipulatives: the abacus and the geoboard, both made of wood and nails. Hundreds were constructed in the 1990s by pre and in-service teachers (Figure 3). Today, various types of abacuses and geoboards, made from different materials and featuring different designs, are readily available. In 2009, as part of the *Patterns* project, we designed a magnetic *Ten Frame* with circular magnetic discs, which aimed to improve students' mental computation, through visualization of the arrangements of the discs in the frame, an essential prerequisite for the development of pre-algebraic concepts. This material was very popular because of its utility and versality. Its magnetic design allows the teacher to show the frame to the entire class without the disks falling, while students can also share their work with the class by raising the frame. Additionally, over the years, several booklets have been published to support the different subjects across different courses. One publication stands out, *Manipulative Materials* (Figure 3), which despite being over 20 years old, remains a valuable and frequently consulted resource.

Figure 3: Some LEM didactical materials

Fonte: Authors

The production of educational resources within the LEM has decreased over time, largely because of the increasing availability and diversity of materials on the market and the gradual replacement of physical resources by digital alternatives. More recently, in 2023, the acquisition of a 3D printer, introduced new opportunities for resource creation. Students of the Mathematics and Natural Sciences Master's course have been using this technology to design and build different artifacts as products of authentic STEAM problems, using the Engineering Design principles. Figure 4 illustrates some of these materials: the first two images depict the construction of a boat, while the remaining four illustrate the development of well-known teaching materials adapted through 3D printing.

Figure 4: Some concrete 3D materials

Fonte: Authors

Both digital and non-digital materials available at the LEM play a crucial role in supporting the mathematical practice conducted in this space. These resources should be used according to the specific objectives and contexts, to ensure comprehensive coverage of all learning contents, accommodating diverse learning styles, and develop a wide range of skills among learners.

LEM - a space for research and dissemination

Another key objective of the LEM is to design and support research in the field of mathematics education. According with the stated goals, the LEM aims to: “b) Design and support research and intervention projects that contribute to advancing knowledge in the field of mathematics education, particularly in the didactics of mathematics, through the efforts of the group of mathematics educators’ teachers of ESEVC” (Vale *et al.*, 2022, p. 2). Also, the research lines developed within the LEM are created to address identified needs and are framed around the areas of interest of its members. These lines are closely linked to (initial) teacher training and the scientific interests of ESEVC. The focus of these research efforts is situated within the context of mathematics education, emphasizing three key areas: Education and Teacher Training, Didactics of Mathematics, and Technology in Mathematics Education (Vale *et al.*, 2022).

As we have observed, the LEM plays an important role in the training of (future) mathematics teachers, while also serving as a space for research. It is within this space that future teachers come into contact with the first requirements of how to carry out an empirical study, culminating in the development of their final coursework. On the other hand, the authors - who are both teachers and researchers of their own practices - identify problems to be studied, with the LEM serving as a context for experimentation, study, and data collection, and its users as active participants. In the next section, we will illustrate some of these links between practice and research. Furthermore, as previously mentioned, the LEM also functions as a space for disseminating mathematical knowledge and emerging trends. This dissemination occurs through various events held within the LEM (e.g., exhibitions, conferences, seminars, workshops, projects). The LEM serves also as the meeting point for the institution's mathematics education group, where educators/researchers collaborate and engage in professional development and research activities.

We can highlight two specific events that took place at the LEM, illustrated through the following images (Figure 5). One example refers to the kick-off meeting of the MaSCE^3 project team (financed by Erasmus+), focused on *Math Trails in School, Curriculum and Educational Environments of Europe*, using the free MathCityMap (MCM) app. The same figure also captures a multiplier event of the MaSCE^3 project, which involved a teacher training session aimed at equipping teachers with the necessary skills to effectively use the (MCM) app at ESEVC. The LEM played a crucial role, providing a dedicated space and resources for collaborative learning and experimentation.

Figure 5: Project MaSCE^3

Source: Authors

Figure 6 illustrates a workshop on STEAM Education conducted as part of the Erasmus+ Blended Intensive Program *Supporting curriculum through integrated STEAM Education practices*. During this week-long training, participants engaged in different activities developed within the mathematics laboratory. The LEM's role was instrumental in providing a space where future teachers could experiment with and implement innovative teaching strategies, fostering an environment conducive to active learning and interdisciplinary integration, through hands-on and technological experiences.

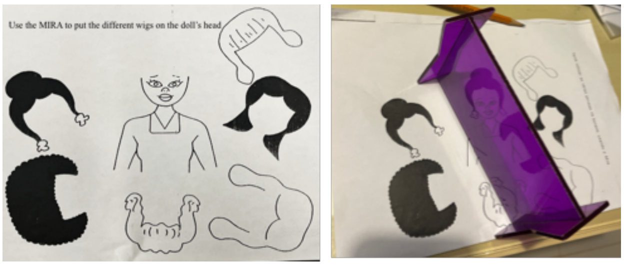
Figure 6: Erasmus+ Blended Intensive Program

Source: Authors

Both events highlight the active involvement of the mathematics education group in internationalization efforts. The first event brought together researchers, while the second included both researchers and (future) mathematics teachers. Participants in these events represented different educational levels and nationalities, including Portugal.

Another objective of the LEM was an initiative initially called "Pedagogical Animation". It aimed to engage students from ESEVC and surrounding schools, along with their teachers, through visits or exhibitions that showcased many of the materials used at the LEM. At the time, these materials were not yet known or used in most schools, so interactive exhibitions were organized with those materials and appropriate tasks where participants could experience them. One of these materials was the Mira or Reflex, a versatile material primarily used for geometric constructions. However, its most popular use was in exploring reflection symmetries, giving it both mathematical and recreational appeal. A particular engaging task involving the Mira, shown in Figure 7, that amused the students, asked to: "Use the MIRA to place the different wigs on the doll's head". This task was not only entertaining but also served as a creative way to introduce mathematical concepts through playful experimentation.

Figure 7: Task with a Mira



Source: Authors

This concern with presenting mathematics as enjoyable, challenging, and accessible continues to be an important aspect that is explored in different situations and aligns with one of the LEM's objectives (Vale *et al.*, 2022):

> k) Organize events (e.g., meetings, seminars, conferences, scientific meetings, exhibitions and other similar initiatives) that make it possible to disseminate, and share the knowledge developed in the area of mathematics education with institutional partners, but also with the general population in order to contribute to building a more positive image of mathematics (p.3)

Figure 8 illustrates some events and exhibitions where the recreational component of mathematics is visible. Also, in various disciplines, students are invited to design and build materials or develop activities where this aspect of the mathematics/challenge/playfulness connection is more visible, such as manipulatives, problems, competitions, challenges, curiosity riddles, charades, etc.

Figure 8: Exhibitions & Challenges

Source: authors

In many institutions, such as elementary schools, the LEM is often seen as a space of enjoyment for students. They associate it with special activities or events, something they don't normally use or do, such as Math Day, where they can engage with materials and experiences that are not part of their everyday classroom routine. However, this recreational aspect should be integrated into regular math classes whenever possible, especially with younger students. The LEM is an adequate setting for fostering this approach. This idea aligns closely with the concept of integrating hands-on and minds-on activities. If the tasks proposed are presented as a challenge or a game rather than traditional work, students will be more receptive to accepting what the teacher proposes. We must contribute for a more positive image of mathematics (Silva, 2014; Vale; Barbosa, 2023b) and show students that mathematics is not difficult or reserved only for someone gifted; it can be fun, challenging and useful.

LEM - a space for teaching, learning and research

As mentioned above, one of the primary missions of the LEM is to support initial teacher training. This includes facilitating classes within the scope of Didactics of Mathematics, which are integrated into the curricula of the different courses offered at ESEVC, as well as hosting different training sessions and courses. As a result, the LEM is mostly used for teaching a wide range of classes across these programs.

To illustrate the LEM's role as a privileged space for active learning and teaching, we selected two tasks, "The heart" and "The tower with bucket". These tasks were designed to highlight how the LEM fosters an environment where the students, future teachers, can be intellectually, socially and physically active (Edwards, 2015; Vale; Barbosa, 2023a). This type of engagement not only enriches the learning experience of students but also provides teachers with valuable opportunities to conduct research on their own practice. Following the description of each task, we present some of the research that directly occurred from their implementation.

The different tasks proposed in the LEM allow students to get to grips with the different types of knowledge they need to acquire (Ponte, 2014; Shulman, 1986) and also their different styles of thinking (Borromeo-Ferri, 2013). It is worth noting that many students learn mathematics through the process of learning how to teach it. This is particularly significant given that not all future teachers are mathematics majors, highlighting the importance of designing tasks that support both their pedagogical and content knowledge.

Paper folding – The heart

"The heart" is a task in the scope of the manipulative materials, where a sheet of paper is used. In particular, paper folding can be a dynamic, creative and appealing strategy to approach several concepts in the mathematics class, facilitating visualization and problem solving (Vale; Barbosa, 2023a). Also, movement associated to the actions performed on the paper - hands-on activities – combined with reflective thought - minds-on activities – allow students to analyze existing relationships and/or experiment with different possibilities until they reach a solution or complete the activity (Vale; Barbosa, 2017, 2023a; Vale *et al.*, 2020). Communicating an idea clearly to another person requires the organization of concepts and thoughts. However, this objective is not always achieved when communication is confined to just one of its aspects, particularly verbal communication. Teachers need to be aware of the potential ambiguity sometimes inherent to communication, especially when spoken language is involved and use multiple sources of information, which may be related, in order to help clarify an idea and address students' different ways of thinking (Krutetskii, 1976). The selected task (Figure 9) involves oral communication, focusing on listening (without seeing) and aims to develop communication without seeing, doing a folding by listening only to the pair's instructions.

For this task, the teacher provides each student with a strip of paper. Students then pair up, deciding who will be the receiver and the transmitter. The transmitter leaves the room with the teacher, where they observe him/her folding the rectangular strip of paper into a heart. The teacher does not provide any information about the folding, but demonstrates the folding process as many times as necessary for the students to understand and memorize it. They return to the LEM and back-to-back, each transmitter gives the instructions to their receiver so that they can do the desired fold. It is important to note that the transmitter has no visual access to what the receiver is doing. Figure 9 illustrates some of the moments of interaction between pairs, capturing the transmitters' efforts to convey instructions and the receivers' attempts to recreate the heart fold.

Figure 9: Paper folding task

Source: Authors

This paper folding task allows to explore and refine various geometric concepts. However, not all students use proper geometric terminology, with some experiencing significant difficulty in articulating their instructions using appropriate mathematical language (e.g., instead of saying pentagon, they say little house). Such incomprehensible or incorrect information often leads to misinterpretation by the receiver, resulting in improper folds, and, consequently, an inaccurate final outcome (as seen in the last image of Figure 9). During the task, it was observed that, in general, students describing the fold often felt the need to use gestures to complement their verbal explanations - even though their receivers could not see them. Some students relied on gestures more heavily than others (as illustrated in the last two images of the first row in Figure 9). These gestures became thinking tools, aiding the transmitters in organizing their reasoning when they lacked the right words to express their ideas.

This task was considered difficult by all the students, mainly because they could not see what their peers were doing during the paper folding process and had to rely only on verbal instructions. They recognized the need to have mathematical knowledge and vocabulary to support their ideas; without these, it can be an obstacle to the communication process. This task is one of many paper-folding activities designed to promote "math talk," emphasizing the critical role of communication in learning mathematics. It demonstrates how effective communication can directly influence understanding, engagement, and the successful completion of tasks. Paper folding has been a valuable teaching tool for these future teachers, not only because it is an easily accessible manipulative material (a sheet of paper) but also because of the different connections it allows us to establish, such as from mathematics to ludic, geometry to numbers, problem-solving to creativity.

In the scope of this work we had the opportunity to develop some studies that allowed us to share our findings. (Barbosa; Vale, 2018; Vale; Barbosa, 2017; Vale *et al.*, 2020).

Authentic Problem– The tower with bucket

The task "The tower with bucket" (Figure 10) is part of a set of tasks used in an ongoing study within the scope of STEAM education, where we used authentic problems solved through the principles of Engineering Design (ED). Authentic or real problems are defined as those found in a real context, which facilitate the establishment of connections with reality (Vale; Barbosa, 2023b, 2024).

A STEAM approach emerges in the context of active learning, involving teaching subjects in an interdisciplinary approach, building on content knowledge developed within and across (some of) these disciplines (Sciences, Technology, Engineering, Arts, Mathematics). We use authentic problems with multiple-solutions, that require hands-on/minds-on activities, leading to active learning. ED is a structured process to address and solve authentic problems, encouraging creativity, collaboration, decision-making, critical thinking, communication and reasoning. This process bridges mathematical and scientific concepts, as well as the arts, through a seven-step cyclical approach (Problem; Imagine; Design; Build; Test & Evaluate; Improve; Solution) promoting STEAM integration (Barbosa *et al.*, 2024; Cunningham & Hester, 2007; Edwards, 2015; Vale & Barbosa, 2023b).

The learning environment within Math Labs is ideal to foster collaboration, exploration, and inquiry-based learning. Based on the ideas from the social constructivism of Vygotsky's (1996) learning is a social process, and thus, collaborative learning spaces can enhance understanding. On the other hand, they advocate for environments that support active learning through manipulatives and technology, allowing students to engage with mathematical concepts in a hands-on manner. Figure 10 illustrates these ideas through the problem "The tower with bucket".

Figure 10: An Authentic Problem

The tower with bucket Build a tower that is tall and sturdy enough to hold a bucket full of pieces. The bucket must not touch the table (minimum: 3cm from the table). The tower support must be stable and hold the full bucket (minimum: bucket weighing about 14 grams).	

Source: authors

The proposed tasks involve the use of materials and actively stimulate student's creativity. These authentic problems are by excellence a laboratory task, in the literal sense of the term, to be used in the LEM because they demand that students be engaged on the activity, using the different resources. To solve this problem students had to use various materials, some provided by the teacher (one paper coffee cup; 1 meter of string, twelve soft drink straws; one roll of adhesive tape; cubes (or coins)) and additional materials available in the LEM (e.g., scale, weights, ruler, paper/notebook, scissors). The task also demands different actions such as cutting, measuring, weighing, testing, and encourages collaboration and movement around the space.

In summary, the students' group work (3-4 elements) can be observed as they progress through the seven phases of the ED (Vale *et al.*, 2023, adapted from Cunningham; Hester, 2007) (Figure 11). The last phase of the ED process (Figure 11 – Share solutions) culminates with the results obtained, in this case the construction of the product. To share their work, we usually associate this last phase with the use of a Gallery Walk (GW) (Vale; Barbosa, 2021), where students construct a poster to explain the entire ED process and present the built prototype. Through the GW the students also have the opportunity to present their work orally and comment anonymously the work of their peers.

Figure 11: The problem through the phases of ED

Source: Authors

This proposal is not usually presented to students, which initially surprise them but also caused enthusiasm. They became collaboratively involved in solving the task. The activity was well accepted by all, not only because of its novelty, but also because it challenged them in ways that extended beyond their usual tasks.

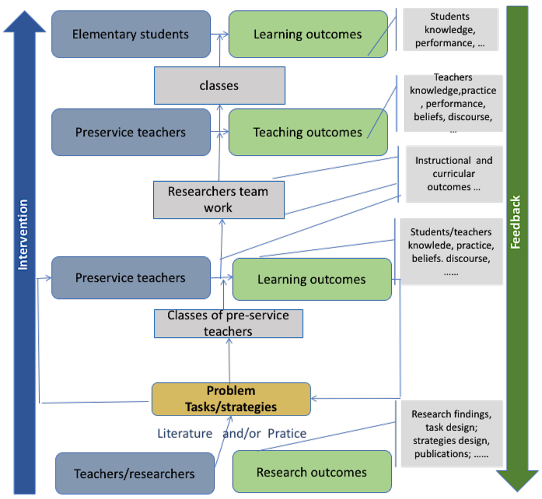
This approach was an opportunity for the future teachers to realize the presence of the various STEAM disciplines through the integration of engineering principles and occasional use of technology. More importantly, it encouraged students to think outside the box when solving authentic problems, giving creative thinking a chance. Although this task could be implemented in a regular classroom, conducting it in the laboratory was an advantage. The space and the resources used and available allowed them to move around freely whenever they needed to, either to exchange ideas or to look for material, as well as to request the presence of the teachers whenever they had doubts. They had the opportunity to fail, but they also had the opportunity to try again and also, to improve their first solutions. On the other hand, teacher researchers could take photos and collect data about the dynamics developed in a friendly environment.

Through this work, we had the opportunity to develop (ongoing) studies that allow us to share findings related to tasks like this. These authentic problems often used common materials but also incorporated technology, such as computer software and 3D printers (Barbosa *et al.*, 2024; Vale *et al.*, 2023).

In parallel to these classes, the LEM supports the design and implementation of studies, research, and intervention projects that contribute to advancing knowledge in the area of mathematics education, particularly in didactics of mathematics. These projects emerge directly from the practice developed during training. Within this framework, teaching and learning tasks, along with innovative strategies, are tested, refined, and put into practice. This reciprocal relationship – between observing practice, problems and addressing emerging questions – creates a continuous cycle of improvement. It generates studies that not only enhance teacher training practices but also lead to better classroom experiences for elementary school students. Ultimately, these results flow back into teacher training classes, enriching future teachers' knowledge and capabilities.

The starting point for our work as researchers arises from a combination of the existing literature and experiences of the teachers/researchers or/and from problems identified in practice. These problems frequently relate to tasks or strategies for the learning and teaching of mathematics. Our concern is to contribute to a better preparation of our future mathematics teachers, while keeping in mind that the ultimate purpose of this training is to have more elementary students that like mathematics, the future pupils of these preservice teachers. Achieving this requires teachers who are not only well-prepared and trained in all aspects, but above all who are motivated to teach. To guide and refine our didactic experiences, we adopted a framework, based in a dynamic cycle (adapted from Vale, 2009), which begins with the work of teachers/researchers and extends through to the dissemination of research findings, the research publications (Figure 12).

Figure 12: The dynamic cycle



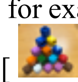
Source: authors

This framework helps us to understand and identify possible flaws and provides opportunities to make improvements in the solutions of our problems. We began by exploring various types of tasks and/or didactical strategies, some of which we developed ourselves and others adapted from literature. These were first tested with our preservice teachers (and, when possible, with in-service teachers) and subsequently implemented with elementary students. Our research consists primarily of qualitative exploratory studies conducted within the scope of the interpretative paradigm. This approach emphasizes studying phenomena in natural settings, with a focus on understanding the problems under study. Data collections methods include observations/interviews, and the analysis of the participants different productions. The tasks/strategies we design are developed in a coherent way to relate the different concepts develop by each task, in the different steps, in order to consolidate the participants' learning (Simon; Tzur, 2004). We follow a dynamic iterative cycle in which we develop, apply, refine, and reapply these tasks and strategies. Each iteration involves reflection and further refinement to ensure that the didactical tools are well-suited to the educational objectives and the needs of the learners.

Some students' perceptions about the LEM

As the LEM is part of the training trajectory of future teachers, it is important to take a brief look at how future teachers, at the end of their training, perceive the LEM, mainly its meaning and usefulness/contribution to them, both as students and as future teachers.
The ideas of these students are very similar among them and generally in accordance with various authors (Anaduaka; Sunday, 2021; Giberti, 2022; Lorenzato, 2006; Silva, 2020).

When asked to define the LEM, students describe it as a space equipped with a variety teaching materials, mainly manipulative, that support mathematical learning and provide opportunities to experience teaching practices. This perception is shared by all students, who highly value the accessibility of resources. For example, whether working in groups or individually, they appreciate knowing that materials, such as books, computers, or even scissors are readily available for them to use. Furthermore, they emphasize the importance of having sufficient time and opportunities to complete their tasks, either individually or in groups. For instance:

> Sometimes I'm solving a problem, …, for example, just the other day I was solving a problem: How many cubes [] will this tower have if it had 10 levels? I couldn't think. I couldn't see what was underneath some of the

> cubes. I went and got the "polycubes" and saw the pattern in the construction of the different levels. Then it was easy. If the class was held in room 10, I wouldn't have had any material, I wouldn't have been able to solve the problem.

It is interesting to note the student's perspectives on the differences between the LEM classes and traditional classes. They refer that "the teaching model is very expository" in traditional classrooms, where students do not have the freedom to move around and access materials when necessary. In contrast, the LEM is perceived as a space for experimentation that encourages both teachers and students to explore new methodologies and approaches, differentiating it from the limitations of a conventional classroom. For the students, the LEM stands out as a more stimulating, flexible, and appropriate environment for 2st century teaching, fostering deep and lasting learning.

When asked whether all mathematics classes should take place in the LEM, their responses were unexpectedly varied. While most students considered the LEM as a valuable space full of resources that enrich teaching and learning, they do not believe it is ideal for all lessons. In concrete, they point out as benefits of the LEM: facilitating access to manipulative materials, making abstract concepts more practical and visual; and providing an ideal environment for activities involving experimentation, reflection, and innovative methodologies. However, they also point out some limitations, stating that some lessons may require a greater focus on theoretical explanations or content that does not depend on manipulative materials, which might be better suited to a traditional classroom. And also, the LEM may not always be sufficient to accommodate all classes or meet specific needs due to logistical or resource constraints. The students suggest that, ideally, the LEM should be reserved for lessons that require experimentation and the use of specific resources, complementing it with traditional classes for more theory-driven or material-independent activities. Only one student described the LEM as a “playful space”, which contrasts with the general perception of the LEM as a placed designed for engaging with tasks not typically encountered in standard classrooms.

Our interpretation it that, although the LEM is essential and significantly contributes to learning, a balanced approach between its use and traditional classes is the best way to ensure a diverse and effective approach. We agree with this perspective, and our practice reflects this balance, diversifying approaches according to content and objectives.

The responses show that students consider the LEM as a space dedicated to learning, experimenting and practicing mathematics. It utilizes a variety of teaching materials, such as games, computer tools, and manipulatives, to facilitate the understanding of mathematical concepts and make teaching more dynamic. It is interesting to note that students contrast the LEM classes with traditional ones, describing the later as following "the teaching model is very expository", lacking the freedom to walk around and access materials when necessary. Thus, for them LEM stands out for providing a more stimulating, flexible and appropriate environment for teaching, fostering deep and lasting learning. In addition, the LEM plays an essential role in the ongoing training of teachers, promoting pedagogical innovation, the integration of theory and practice and the development of methodologies adapted to 21st century teaching. This resource-rich environment also fosters critical thinking, mathematical skills and creativity, and is ideal for playful and meaningful practices.

They were also asked about the greatest contribution the LEM can make to them as learners and as future teachers. In general, they stated that the LEM has an important contribution for them as students and future teachers. They identified several ideas, like: the opportunity to explore diverse teaching materials, often unfamiliar, broadening one's pedagogical repertoire for the classroom; it allows us to observe the concretization of abstract mathematical concepts, enriching learning with practical and interactive approaches that make the content more accessible and engaging; the contact with pedagogical innovation practices, help develop creative strategies that show students how to learn by "playing/experiencing/experimenting", promoting dynamic and stimulating classes; provides practical support mainly for overcoming the resource limitations found in many schools; contributes to their professional development of essential teaching skills, such as the ability to adapt resources to students' needs, foster critical thinking and create more meaningful teaching experiences. Another idea that we highlight it is that the LEM contributes to practical and investigative learning, where theory and practice are integrated. This leads us to infer that, as these students are at the end of their course, they are reflecting more deeply on their role. We can summarize and infer that LEM acts as a space for innovation and preparation, enabling future teachers to promote more effective teaching in line with contemporary demands.

From our perspective as educators and users of the LEM, the practices carried out in this space contribute to creating a unique culture of learning and teaching. This culture emerges from the interconnected relationships between the physical space, materials, technology, task nature, teaching strategies, and users' engagement. The LEM fosters exploration of mathematical concepts through experiential activities and materials, bridging the gap between theory and practice. For us, it enhances teacher training through experiential learning, collaboration, and reflective practices.

However, we are interested to explore the perspectives of others students who are at different stages of their training. Their responses may differ, as they might not yet share the same level of maturity and professional development regarding teaching, learning, and research like these final-course students. This variability would be in accordance with Feiman-Nemser (2001) assertion that perceptions evolve throughout teacher preparation.

Some concluding remarks

This article was an opportunity for the authors to reflect on the evolution and significance of the work developed at the LEM, which has accompanied teacher training courses since the opening of the ESEVC. We explored the various features and benefits of LEM in math education into teacher training programs as essential for enhancing pedagogical practices and improving student outcomes in mathematics.

The reflection required to articulate the ideas expressed here allow us to argue that a Math Lab is a fundamental space in teacher training programs for the integral development of future educators. This includes not only the cognitive dimension but also the social and interpersonal one (Anaduaka; Sunday, 2021; Lorenzato, 2006; Silva *et al.*, 2024). In short, everything related to mathematics and mathematics education must pass through the LEM. Over 35 years of continuous work, we confidently state that establishing the LEM has been one of the most impactful ideas put into practice. The LEM has evolved far beyond its initial purpose in the distant 80's as merely a repository of teaching materials. Today, it serves as a cornerstone for the professional development of teachers involved in the LEM and contributes to the education of students and teachers across various educational levels who have engaged with the institution.

In addition to addressing the challenges that naturally arise in a teacher training institution, the LEM has become an incubator for research problems to be studied, both in the didactics of mathematics and in teacher education. The research questions arise from our practice, reflections about that practice, and the existing literature, particularly regarding the early years of education. We defend the maxim that "no one learns if are not willing to learn". So, to foster motivation the context, the learning environment must be not-threatening, allowing collaboration with peers to ensure success on the tasks carried out. The affective dimension of learning is very important to us, where the challenge, the collaboration, discussions and sharing are essential for students to achieve the maximum of their math potential. The LEM provides a unique context to nurture curiosity and creativity—two fundamental pillars for knowledge construction and meaningful learning. It is a space where individuals with diverse knowledge, experiences, and aspirations come together, unified by a common goal. Within this environment, the LEM offers freedom and opportunities for everyone to progress according to their needs.

One of our core objectives has been to develop tasks and teaching strategies that inspire future teachers to learn and enjoy mathematics. LEM has proven to have this unique capacity. Students often express that when they enter the LEM, their mindset shifts: they are not entering "a classroom," but rather a space where they will "work on mathematics" or where they feel empowered to "access any material they need."

As mentioned earlier, teacher training institutions continue to face significant challenges, highlighting the ongoing need for innovation and development. In light of this, we recommend that all training institutions establish a Mathematics Education Laboratory, as advocated by several researchers (Cabrita *et al.,* 2002; Gilberti, 2022; Hattie, 2009; Lorenzato, 2006; Silva, 2020; *Silva et al.*, 2024). Teacher training programs should incorporate experiences, like those offered by the LEM, that challenge and expand preservice teachers' knowledge and beliefs. These programs should allow future educators to engage with the same tasks and adopt the teaching and learning principles they are expected to apply in their own classrooms. By doing so, teacher preparation can foster the development of innovative, reflective, and highly capable educators who are well-equipped to inspire the next generation of learners.

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References

ANADUAKA, U. S.; SUNDAY, A. O. Mathematics laboratory: practical solution to classroom “mathemagics” in schools. **Continental Journal of Education Research**, v. 13, n. 2, p. 1–8, 2021.

BARBOSA, A.; VALE, I. As dobragens como uma estratégia de aprendizagem ativa. In: ALVES, D.; PINTO, H.; DIAS, I.; ABREU, M.; MUÑOZ, R. (orgs.). *In*: **Atas da VII Conferência Internacional Investigação**

Práticas e Contextos em Educação. Escola Superior de Educação e Ciências Sociais Instituto Politécnico de Leiria, 2018. p. 83-91.

BARBOSA, A.; VALE, I.; ALVARENGA, D. The use of Tinkercad and 3D printing in interdisciplinary STEAM education: a focus on engineering design. **STEM Education**, v. 4, n. 3, p. 222-246, 2024.

BORROMEO FERRI, R. Mathematical thinking styles and their role in learning mathematics. **Educational Studies in Mathematics**, v. 83, n. 3, p. 269-288. 2013.

CABRITA, I.; ARAÚJO E SÁ, M. H.; MARTINS, I. Espaços laboratoriais: unidade na diversidade. *In*: **Actas do XI ENDIPE – Igualdade e diversidade na Educação**, Aveiro, Portugal, 2002. p. 1-21.

CLANDININ, D. J., & CONNELLY, F. M. (2000). Narrative inquiry: Experience and story in qualitative research. San Francisco, CA: Jossey-Bass, 2000.

CLEMENTS, D. H.; SARAMA, J. **Learning and teaching early math: the learning trajectories approach**. New York: Routledge, 2009.

CRESWELL, J. **Research Design: Qualitative, Quantitative, and Mixed Method Approaches**. 3rd Edition, Thousand Oaks, CA: Sage, 2009.

CUNNINGHAM, C. M.; HESTER, K. Engineering is Elementary: An Engineering and Technology Curriculum for Children. *In*: **Proceedings of the ASSE Annual Conference and Exposition,** Honolulu, HI, USA, 1 January 2007; pp. 12.639.1–12.639.18. 2007.

DI SIA, P. The laboratory of mathematics in primary school: A practical approach for understanding and learning. **International Letters of Social and Humanistic Sciences**, v. 44, p. 21-28, 2014.

DITTMER, K. 1971 **Guidelines for developing a mathematics laboratory**. DISSERTATION UNIVERSITY, ALABAMA

EDWARDS, S. Active learning in the middle grades. **Middle School Journal**, n. 46, p. 26–32, **2015.**

FEIMAN-NEMSER, S. (2001). From preparation to practice: Designing a continuum to strengthen and sustain teaching. **Teachers College Record**, v.103, n.6, p. 1013-1055, 2001.

GIBERTI, C. A teacher training project to promote mathematics laboratory during the COVID-19 health crisis in Italy. **European Journal of Science and Mathematics Education**, v. 10, n. 3, p. 256-268, 2022.

JURDAK, M. Contrasting Perspectives and Performance of High School Students on Problem Solving in Real World Situated, and School Contexts. **Education Studies of Mathematics**. V. 63, p. 283–301. 2006.

HATTIE, J. **Visible learning: a synthesis of over 800 meta-analyses relating to achievement**. New York: Routledge, 2009.

KRUTETSKII, V. A. **The psychology of mathematical abilities in schoolchildren**. Chicago: University of Chicago Press, 1976.

LEIKIN, R. Interplay between creativity and expertise in teaching and learning of mathematics. *In*: CSÍKOS, C.; RAUSCH, A.; SZITÁNYI, J. (eds.). **Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education**. PME, 2016. p. 19-34.

LLINARES, S. Instructional quality of mathematics teaching and mathematics teacher education. **Journal of Mathematics Teacher Education**, v. 24, n. 1, p. 1–3, 2021.

LORENZATO, S. **Laboratório de Ensino de Matemática**. Autores Associados, 2006.

MEURER, S.; BORGES, F.; HERMANN, W. The Teaching Laboratory as a formative space for teachers who teach Mathematics. **Ensino em Revista**, v. 30, p. 1-24, 2023.

MILES, M. B.; HUBERMAN, A. M. **Qualitative data analysis**. Sage Publications: Thousand Oaks, CA, USA, 1994.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS. **Principles to Actions: Ensuring Mathematical Success to All**. Reston: NCTM, 2014.

OECD. **Future of Education and Skills 2030**, Report from OECD, 2019. Available from https://www.oecd.org/education/2030-project/teaching-and-learning/learning/learning-compass-2030/OECD_Learning_Compass_2030_Concept_Note_Series.pdf

OLIVEIRA, Z. V.; KIKUCHI, L. M. O laboratório de matemática como espaço de formação de professores. **Cadernos de Pesquisa**, v. 48, n. 169, p. 802-829, 2018.

OME. **Capacity building series: Communication in the mathematics classroom**. Ontario Ministry of Education, Queen's Printer for Ontario, 2016.

PÓLYA, G. **A arte de resolver problemas: Um novo aspecto do método matemático**. Editora Interciência, 1978.

PONTE, J. P. (2014). Formação do Professor de Matemática: Perspetivas atuais. *In* PONTE, J.P (Org.). **Práticas Profissionais dos Professores de Matemática**. Instituto de Educação da Universidade de Lisboa, ProjetoP3M, 2014. p.343-358.

SHULMAN, L. S. Those who understand: Knowledge growth in teaching. **Educational Researcher**, v.15, n. 2, p. 4-14, 1986.

SILVA, A. J. N. **A ludicidade no laboratório: considerações sobre a formação do futuro professor de matemática**. Curitiba, PR: Editora CRV, 2014.

SILVA, A. J. N. O laboratório de educação matemática e a formação inicial de professores de matemática. **Revista Internacional Educon**, v. 1, n. 1, p. 1-13, 2020.

SILVA, A.; CABRITA, I.; VALE, I. First Mathematics Education laboratory in Higher Education in Portugal: stories built with words. **Revista Internacional de Pesquisa em Educação Matemática**, v. 14, n. 2, p. 1-18, 2024.

SIMON, M.; TZUR, R. Explicating the role of mathematics tasks in conceptual learning: an elaboration of the hypothetical learning trajectory. **Mathematical Thinking and Learning**, v. 6, n. 2, p. 91-104, 2004.

VALE, I. Mathematics and patterns in elementary schools: perspectives and classroom experiences of students and teachers. *In*: VALE, I.; BARBOSA, A. (Orgs.). **Patterns: multiple perspectives and contexts in mathematics education**. ESE-IPVC, Projecto Padrões, 2009. p. 7-14.

VALE, I.; BARBOSA, A. Comunicação matemática: a articulação entre ouvir, ver e falar. *In*: PIRES, M. V.; MESQUITA, C.; LOPES, R. P.; SANTOS, G.; CARDOSO, M.; SOUSA, J.; SILVA, E.; TEIXEIRA, C. (Eds.). **Livro de Atas do II Encontro Internacional de Formação na Docência, INCTE 2017**. Bragança: Instituto Politécnico de Bragança, 2017. p. 257-264.

VALE, I.; BARBOSA, A. Promoting mathematical knowledge and skills in a mathematical classroom using a Gallery Walk. **International Journal of Research in Education and Science (IJRES)**, v. 7, n. 4, 2021.

VALE, I.; BARBOSA, A. Active learning strategies for an effective mathematics teaching and learning. **European Journal of Science and Mathematics Education**, v. 11, n. 3, p. 573-588, 2023a.

VALE, I.; BARBOSA, A. Visualization: A Pathway to Mathematical Challenging Task. In LEIKIN, R. (Ed.), **Mathematical challenges for all** (cap.15). Cham, CH: Springer, 2023b p. 283-306.

VALE, I.; BARBOSA, A.; CABRITA, I. Paper folding for an active learning of mathematics: an experience with preservice teachers. **Quaderni di Ricerca in Didattica (Mathematics)**, v. 7, p. 53-59, 2020. ISSN on-line 1592-4424.

VALE, I.; BARBOSA, A.; PEIXOTO, A.; FERNANDES, F. Solving authentic problems through engineering design. **Open Education Studies**, v. 5, n. 1, p. 1-9, 2023.

VALE, I., FONSECA, L., BARBOSA, A. **Laboratório de Educação Matemática** – regulamento e atividades. (texto não publicado). Viana do Castelo: ESEVC-LEM, 2022.

VYGOTSKY, L. S. **A formação social da mente**. São Paulo: Martins Fontes, 1996.