STEM Education

STEM Education, 6(5): 848–884
DOI: 10.3934/steme.2026034
Received: 10 April 2026
Revised: 04 July 2026
Accepted: 08 July 2026
Published: 12 August 2026

https://www.aimspress.com/journal/steme

Research article

Engineering design as a framework for developing pre-service teachers' TPACK in digital making environments

Ana Barbosa[1,2,*] and Isabel Vale[1,3]

[1] Instituto Politécnico de Viana do Castelo, Rua Escola Industrial e Comercial Nun'Álvares, 34, 4900-347, Viana do Castelo, Portugal; anabarbosa@ese.ipvc.pt, isabel.vale@ese.ipvc.pt
[2] inED, Centro de Investigação e Inovação em Educação, Instituto Politécnico de Viana do Castelo, Viana do Castelo, Portugal
[3] CIEC, Research Centre on Child Studies, University of Minho, Braga, Portugal

* **Correspondence:** Email: anabarbosa@ese.ipvc.pt; Tel: +351-258-806-200.

Academic Editor: Xuesong Zhai

Abstract: The integration of digital making technologies and engineering design (ED) in teacher education has created new opportunities to support the professional knowledge required for teaching in technology-rich environments. Within this context, the TPACK framework provides a valuable lens for understanding how content, pedagogy, and technology are dynamically integrated in practice. However, less is known about how these domains of knowledge are enacted and interconnected throughout iterative design processes. This study explored how the ED process supports the development and integration of pre-service teachers' TPACK in digital making environments. A qualitative interpretative study was conducted with 12 pre-service teachers enrolled in a master's program in Mathematics and Science Education at a Portuguese higher education institution. Multiple data sources were collected, including classroom observations, participants' reflective journals, sketches, and digital and physical artifacts produced using Tinkercad and 3D printing. The findings indicate that the ED process used to design mathematical manipulatives created opportunities for participants to mobilize and progressively integrate content, pedagogical, and technological knowledge. Different levels of knowledge integration emerged across the steps of the design cycle, with iterative testing and redesign playing a key role in fostering reflective decision-making and professional learning. Overall, the study highlights the potential of ED and digital making as promising pedagogical frameworks for supporting integrated professional knowledge and fostering innovative practices in teacher education.

Keywords: STEAM education, engineering design, digital making, math manipulatives, TPACK, teacher education

1. Introduction

The increasing presence of digital technologies in education has transformed the knowledge and competencies expected of teachers in contemporary classrooms. In particular, the integration of digital tools into teaching and learning has created new demands for teachers to combine disciplinary knowledge, pedagogical reasoning, and technological competence in ways that support meaningful learning experiences [1,2]. Within this context, teacher education programs are increasingly challenged to prepare future teachers to design learning environments that integrate content, pedagogy, and technology in coherent and purposeful ways. The Technological Pedagogical Content Knowledge (TPACK) framework has therefore become a central reference for understanding the knowledge required for effective teaching in technology-rich environments, emphasizing the dynamic interaction among these domains rather than their separate application [1,3].

At the same time, current educational agendas increasingly emphasize the importance of preparing learners to address authentic and complex problems through interdisciplinary knowledge, creativity, collaboration, and critical thinking [4–6]. In response to these demands, design-based and technology-enhanced learning environments have gained relevance in educational research and practice, particularly within STEAM education [7–9]. Among these approaches, digital making environments have emerged as promising contexts for fostering problem solving, spatial reasoning, and conceptual understanding through the creation of tangible artifacts [10–12].

Within this perspective, the engineering design (ED) process has been widely recognized as a pedagogical framework capable of supporting knowledge integration across disciplines. The iterative nature of the ED cycle encourages learners to analyze constraints, generate and refine ideas, and justify their decisions over time [8,13–15]. In teacher education contexts, engaging pre-service teachers in design-based activities mediated by digital tools provides a particularly valuable setting for investigating how professional knowledge is enacted, mobilized, and integrated through action, experimentation, and reflection [15–17].

Recent studies have shown that the use of Tinkercad and 3D printing can foster spatial reasoning, problem solving, and the creation of customized manipulatives for mathematics education, while encouraging reflection on their educational use [11,12,16,18]. However, less is known about how these domains of knowledge are integrated and enacted throughout iterative design processes such as the ED cycle. Understanding how knowledge integration unfolds across design phases is essential for conceptualizing TPACK as a dynamic and practice-based construct in teacher education [1,19,20].

This study builds on previous work conducted in teacher education and digital making contexts [16,18]. Earlier studies showed that engaging pre-service teachers in the ED process supported the development of technological skills, conceptual understanding, and positive perceptions of digital fabrication technologies. The present study extends this line of research by moving beyond the identification of knowledge and skills to examine how TPACK is enacted and integrated throughout the design process. While previous studies documented participants' performance and perceptions across the ED cycle, the current investigation provides a more fine-grained analysis of the dynamic relationships among content, pedagogy, and technology. Thus, the purpose of this study is to understand how the engineering design process supports the development and integration of pre-service teachers' TPACK in digital making environments. To achieve this aim, the study addresses the following research questions: 1) How is TPACK enacted across the steps of the engineering design process? 2) What levels of knowledge integration emerge during the design and production of 3D-printed manipulatives?

2. Theoretical framework

2.1. TPACK in teacher education

The quality of teaching has long been associated with the knowledge teachers possess and apply in practice. Research in teacher education consistently emphasizes that teaching is a complex professional activity requiring specialized forms of knowledge that extend beyond general pedagogical skills. Effective teaching depends on a knowledge base that integrates subject matter understanding, pedagogical reasoning, and knowledge of learners and contexts [1,21]. Teaching is therefore understood as a knowledge-based profession in which teachers mobilize multiple domains of expertise to interpret classroom situations and design meaningful learning experiences [21,22].

A major milestone in the conceptualization of teacher knowledge was the work of Shulman [21,23], who proposed a framework describing the knowledge base for teaching. Shulman identified several categories of teacher knowledge, including content knowledge, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge, knowledge of learners, knowledge of educational contexts, and knowledge of educational ends, purposes, and values. Among these, pedagogical content knowledge (PCK) became particularly influential, referring to the specialized knowledge that enables teachers to transform subject matter into forms understandable to learners. Effective teaching requires an understanding of how particular topics are organized, represented, and adapted to students' needs and prior knowledge [21,22]. These categories emphasize the multidimensional nature of teaching and provide an important conceptual foundation for later models of teacher knowledge integration. Table 1 summarizes the categories of the knowledge base for teaching by Shulman.

Table 1. Categories of the knowledge base for teaching (adapted from Shulman [21,23]).

Categories of teacher knowledge	Description
Content knowledge	Knowledge of the subject matter itself, including concepts, structures, and ways of organizing knowledge within a discipline.
General pedagogical knowledge	Knowledge of general principles of classroom management, organization, and instructional strategies that are not specific to a particular subject.
Curriculum knowledge	Knowledge of the curriculum, materials, and resources available for teaching particular subjects and topics at different levels.
Pedagogical content knowledge	Knowledge of how to represent and transform the subject matter in ways that make it understandable to learners, including awareness of common misconceptions and effective representations.
Knowledge of learners and their characteristics	Knowledge of students' cognitive, developmental, and motivational characteristics, including their prior knowledge and learning needs.
Knowledge of educational contexts	Knowledge of the classroom, school, and broader social and cultural contexts in which teaching and learning occur.
Knowledge of educational ends, purposes, and values	Understanding of the aims of education and the values that guide teaching decisions and educational practice.

Building on Shulman's work, research in mathematics education has emphasized that teaching mathematics requires specialized forms of professional knowledge that combine conceptual understanding, attention to students' thinking, and instructional decision-making [22,24,25]. In particular, the construct of Mathematical Knowledge for Teaching (MKT) highlights that mathematics teaching involves forms of knowledge specific to instructional contexts, reinforcing the importance of integrating disciplinary and pedagogical dimensions [22]. Although the present study is primarily framed within another framework, these perspectives provide relevant background for understanding the specific demands of mathematics teacher education.

The rapid expansion of digital technologies in education has further transformed the knowledge required for teaching. The integration of digital tools into classrooms has introduced new demands on teachers, requiring them to understand not only content and pedagogy but also the affordances and constraints of technological tools. In response to these developments, Mishra and Koehler [1] proposed the concept of Technological Pedagogical Content Knowledge (TPACK) as a framework for describing the knowledge required for effective technology integration in education. The TPACK framework conceptualizes teacher knowledge as the interaction among three core domains: content knowledge (CK), pedagogical knowledge (PK), and technological knowledge (TK). At the intersections of these domains, additional forms of knowledge emerge, including technological content knowledge, technological pedagogical knowledge, and technological pedagogical content knowledge, which represents the integrated understanding required to design meaningful learning experiences with technology [1,2].

The integrative nature of teacher knowledge in technology-rich environments is represented in Figure 1 [3], which illustrates the relationships among technological, pedagogical, and content knowledge within the TPACK framework. As shown in Figure 1, effective teaching with technology depends not only on possessing knowledge in each domain but also on understanding how these domains interact and support one another in practice. In this sense, TPACK is not simply the sum of technological, pedagogical, and content knowledge; rather, it is an emergent form of knowledge that arises from the dynamic interaction of these domains in specific teaching contexts [1,3].

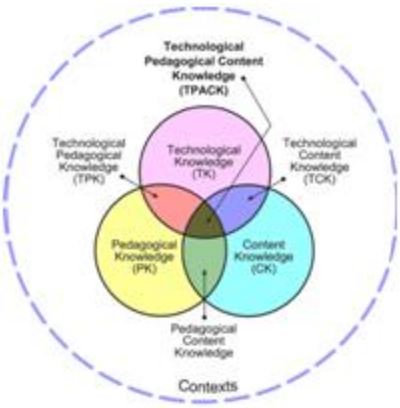

Figure 1. Components of the TPACK framework [3].

While the TPACK framework conceptualizes teacher knowledge through interconnected domains (CK, PK, TK, and their intersections), several authors have emphasized that the enactment of TPACK is dynamic and context-dependent rather than static, requiring attention to how knowledge domains are coordinated in practice and across tasks [1–3].

Over the past two decades, the TPACK framework has become one of the most widely used conceptual models for understanding teacher knowledge in technology-rich environments. The framework has been applied across disciplines and educational contexts to investigate how teachers develop the competencies required to integrate digital technologies into teaching and learning. In mathematics education, TPACK has been recognized as particularly valuable because it provides a comprehensive lens for examining the synergy between technology, pedagogy, and mathematical content, which is essential for effective teaching in contemporary classrooms [26,27]. Research has shown that teacher education programs that explicitly address the relationships among technology, pedagogy, and content can support the development of teachers' competencies and confidence in using technology effectively, leading to improved instructional practices and more meaningful integration of digital tools in mathematics teaching [2,26].

Despite the extensive use of the TPACK framework in teacher education, recent research highlights the need for studies that move beyond the measurement of knowledge components to examine how teacher knowledge is enacted in practice. In particular, scholars have emphasized the importance of understanding the dynamic and contextual nature of TPACK, including how knowledge is integrated and applied in authentic teaching situations and how it evolves through experience and reflection [19,20]. These perspectives suggest that TPACK should be understood not only as a set of competencies but also as professional knowledge enacted through authentic tasks involving design, experimentation with technologies, pedagogical decision-making, and reflection. This perspective is particularly relevant in digital making contexts, where pre-service teachers must coordinate mathematical content, pedagogy, and technological affordances throughout iterative design processes.

Recent studies have further expanded the understanding of TPACK in pre-service teacher education, emphasizing its dynamic, situated, and evolving nature in response to increasingly complex digital environments. For instance, Gonscherowski and Rott's [28] systematic review of TPACK instruments used with pre-service teachers highlights the continued relevance of the framework while also pointing to the need for approaches that capture how knowledge is enacted in authentic and technology-rich contexts. In parallel, emerging studies have explored the intersections between TPACK and artificial intelligence, proposing extensions such as Intelligent-TPACK to conceptualize the knowledge required to integrate AI-based tools into teaching practices [29,30]. These perspectives reinforce the view that teacher knowledge should not be understood as a static set of competencies but rather as an adaptive and context-sensitive process shaped by evolving technological affordances and pedagogical demands.

2.2. Engineering design as an integrative framework

In recent decades, STEAM education has gained increasing attention in educational research and policy as a response to the growing complexity of societal and technological challenges. Contemporary educational agendas emphasize the need to prepare learners to address real-world problems through interdisciplinary knowledge, creativity, and innovation, highlighting the importance of developing competencies such as problem solving, collaboration, and critical thinking [4–6].

Within this perspective, STEAM education has emerged as a framework that emphasizes the purposeful integration of science, technology, engineering, arts, and mathematics through authentic learning experiences. Rather than treating disciplines as isolated domains, integrated approaches seek to connect knowledge across fields in ways that reflect real-world practices and support deeper learning [8,9]. In this context, authentic problems play a central role, requiring learners to mobilize knowledge and skills from multiple disciplines to develop meaningful and viable solutions [7,31]. Frameworks of STEAM integration have been conceptualized along a continuum ranging from disciplinary and multidisciplinary approaches to more advanced forms of interdisciplinary and transdisciplinary integration, in which knowledge and practices from different domains are coordinated to address complex real-world problems [32]. Achieving higher levels of integration requires learning environments that support collaboration, inquiry, and design-based activities, enabling learners to connect theoretical knowledge with practical application [8,9,13]. In this sense, engineering has been increasingly recognized as a discipline capable of linking knowledge across domains and providing a context for integrating scientific, technological, and mathematical ideas through purposeful design activities [13,14,31].

Among the approaches proposed to support interdisciplinary integration, the engineering design (ED) process has been widely recognized as a central framework for organizing learning in integrated STEAM contexts. ED provides a structured yet flexible framework for addressing complex problems, enabling learners to generate ideas, test solutions, and refine their designs through iterative processes. Studies have shown that integrating the engineering design process into STEAM learning environments can support design thinking and problem-solving abilities, particularly when learners engage in activities that involve modeling, testing, and evaluating solutions in authentic contexts [15–17].

A defining characteristic of the ED process is its iterative nature. Unlike linear problem-solving approaches, the ED cycle involves repeated steps of planning, testing, and redesign, enabling learners to learn from failure and improve their solutions over time [14]. This iterative process supports reasoning, reflection, and decision-making skills, as learners analyze constraints, evaluate alternatives, and justify their choices. Research indicates that ED tasks encourage students to revisit earlier stages of the process, reconsider assumptions, and refine their ideas, reinforcing the dynamic nature of problem solving and knowledge construction [7,8,13,31].

The incorporation of engineering into educational contexts gained momentum with initiatives aimed at introducing engineering concepts to learners from an early age. A key contribution was the work of Hester and Cunningham, who proposed a simplified model of the engineering design process to support problem-solving through hands-on and integrated learning experiences. This model conceptualizes ED as a five-step iterative and flexible approach that enables learners to address authentic problems while connecting knowledge across disciplines, laying the foundation for subsequent adaptations of the ED cycle in educational settings [14].

Building on this model, several researchers have adapted the ED cycle to align more closely with educational goals and learning contexts. Vale and colleagues [15] developed an adapted version of the ED cycle to make the iterative nature of the design process more explicit and accessible to learners, particularly in teacher education settings. The adapted cycle maintains the core logic of the original model while introducing clearer terminology and emphasizing the possibility of revisiting previous steps in response to testing and evaluation outcomes. In this study, we adopted the model proposed by Vale et al. [15], which consists of seven interconnected steps (Figure 2): problem, imagine, design, (re)build, (re)test and evaluate, redesign, and solution. The model explicitly incorporates the possibility of redesigning and rebuilding the artifact whenever necessary, after testing and evaluation, reinforcing the iterative nature of the process.

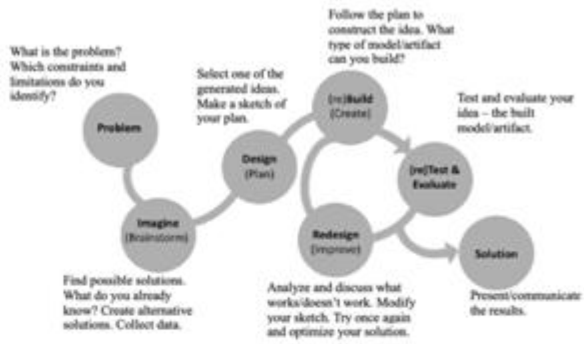

Figure 2. ED cycle adapted from Hester and Cunningham [14] and proposed by Vale et al. [15].

The adapted ED cycle emphasizes the cyclical and iterative nature of problem solving, allowing learners to revisit earlier stages whenever necessary to improve their solutions. This structure supports the development of metacognitive skills, as learners are encouraged to reflect on their decisions, justify their design choices, and communicate their results. In educational contexts, the inclusion of explicit stages for evaluation and redesign reinforces the understanding that learning emerges through experimentation, reflection, and continuous improvement. Given these features, the ED process provides a particularly relevant framework for teacher education.

2.3. Design-based learning through digital making

Digital making environments have emerged as powerful contexts for learning by enabling students to design, create, and refine artifacts that represent ideas and concepts in tangible forms. Rooted in constructionist perspectives on learning, these environments emphasize active engagement in the design and production of artifacts as a way of developing understanding through meaningful action and reflection [10,33]. The integration of digital fabrication technologies in educational settings has expanded opportunities for learners to explore problems, test solutions, and make connections between theoretical knowledge and real-world applications, particularly within STEAM education contexts [10,16,34,35]. By engaging in cycles of designing, building, and evaluating artifacts, learners participate in authentic problem-solving processes that foster deeper conceptual understanding across disciplines.

A defining feature of digital making environments is their potential to support learning through the creation of tangible artifacts that embody ideas and concepts, enabling learners to move between digital representations and physical objects. The combined use of computer-aided design (CAD) and 3D printing allows learners to design, test, and refine models through iterative cycles of experimentation, fostering problem-solving, spatial reasoning, creativity, and conceptual understanding [11]. In mathematics education and teacher education contexts, the creation of digital and physical models has been shown to support the visualization of abstract concepts and the exploration of relationships between shapes, dimensions, and structures, facilitating meaningful connections between theory and practice [11,12,16,18,36]. Research further indicates that designing and printing customized artifacts, in particular manipulatives, enables teachers and learners to adapt resources to specific instructional goals and to represent mathematical relationships in ways that extend beyond two-dimensional diagrams or symbolic representations [16,35]. These possibilities expand opportunities for innovation in teaching and learning, while also strengthening the integration of disciplinary knowledge, technological skills, and pedagogical practices.

In teacher education, the use of 3D modeling and printing has been increasingly associated with the development of professional competencies involving technological, pedagogical, and content knowledge. Research highlights the importance of preparing future teachers to use these technologies meaningfully in educational contexts, rather than focusing solely on technical skills. The integration of 3D modeling and printing into teacher education programs has been shown to contribute to technological confidence and self-efficacy, while also encouraging reflection on pedagogical strategies and instructional design [12,37]. In particular, implementing these technologies within structured learning experiences can enhance skill development, engagement, and hands-on learning, although challenges related to resource availability, time constraints, and curriculum integration remain significant in many educational settings, highlighting the need for teacher education programs that combine practical experience with pedagogical guidance [16,34].

CAD environments play a crucial role in digital making by enabling the creation of virtual representations of physical objects and supporting the exploration of design solutions before fabrication. Among the available tools, Tinkercad has become one of the most widely used platforms in educational contexts due to its accessibility, intuitive interface, and suitability for beginners, particularly in teacher education programs. As a web-based environment with drag-and-drop functionalities, Tinkercad allows users to design, manipulate, and refine three-dimensional models without requiring advanced technical expertise, making it especially appropriate for pre-service teachers who are developing their technological competencies [11,16,18,36]. Working with CAD tools supports the development of spatial visualization and geometric reasoning, as users must consider dimensions, proportions, and spatial relationships when constructing digital models. These experiences also promote iterative thinking, as learners experiment with different configurations and evaluate the consequences of their design decisions, aligning closely with the principles of the ED process [11,16,18].

Beyond technical skills, digital making environments support the development of broader competencies relevant to contemporary education. Engaging in design and fabrication activities encourages learners to integrate knowledge from different domains, communicate ideas effectively, and collaborate with peers to solve complex problems. Digital fabrication technologies can foster creativity, critical thinking, and adaptability, particularly when learners are engaged in authentic design challenges requiring the application of disciplinary knowledge in new situations [10,36,38]. These competencies are particularly relevant in teacher education, where future educators must be prepared to design learning experiences that integrate content knowledge, pedagogical strategies, and digital technologies.

Despite their potential, the integration of digital making technologies into teacher education also presents challenges. Studies consistently highlight issues related to equipment access, time required for learning and implementation, and the need for pedagogical support. The complexity of managing digital fabrication processes and aligning them with curriculum requirements can create uncertainty for teachers who are unfamiliar with these technologies. In addition, the successful integration of digital making into educational practice depends on the availability of training opportunities that support the development of both technical and pedagogical competencies [16,36,37]. Research suggests that effective teacher education programs should provide sustained opportunities for hands-on practice, collaborative learning, and reflection on teaching strategies, enabling future teachers to integrate digital fabrication technologies in ways that enhance learning rather than simply adding technological novelty [37–39].

3. Methodology

3.1. Research design

This study adopted a qualitative research methodology, with an interpretative design [40], aimed at understanding how the ED process supports the development and integration of pre-service teachers' TPACK in digital making environments. Qualitative research is particularly appropriate when the goal is to explore participants' experiences, actions, and meanings in naturalistic contexts and to interpret how knowledge is constructed through interaction with tasks and learning environments [40,41]. Rather than measuring outcomes, the study emphasized understanding the processes through which knowledge was enacted and transformed throughout the iterative phases of the ED cycle, while designing and producing mathematical manipulatives using digital tools.

Participants were 12 pre-service teachers enrolled in a Master's degree in Mathematics and Science Education at a Portuguese higher-education institution. These participants were future teachers of children aged 5–12 years old. The group included 11 women and 1 man, with an average age of 22 years. Prior to enrolling in the master's program, these pre-service teachers had completed a Bachelor's degree in Basic Education, with a duration of six semesters. This undergraduate program included coursework related to didactics, general education, and content knowledge development in formal and non-formal educational contexts. The master's program they were attending grants professional teaching qualifications with a specialization in Mathematics and Science, providing more focused preparation in these subjects from both scientific and didactic perspectives.

The study adopted a convenience sampling strategy, as participants were recruited from a naturally occurring educational context in which the didactical experience was implemented. Specifically, all students enrolled in the Master's degree where the intervention took place were invited to participate. Thus, the sample was not purposively selected according to predefined criteria but rather reflected the complete cohort of pre-service teachers attending the course during that semester. These participants were considered particularly relevant to the aims of the study because they were future teachers preparing to teach mathematics and science to children aged 5–12 years and were simultaneously engaged in the same learning experience, under similar pedagogical conditions and over an extended period of time.

During the study, participants were enrolled in a Didactics of Mathematics course, taught by the researchers, which served as the context for the didactical experience and data collection. The work developed during the semester was aligned with current curricular guidelines for the teaching and learning of mathematics and emphasized topics like STEAM education, problem solving, reasoning, mathematical communication, and the use of manipulatives, supported by principles of active learning and technology integration. Because the researchers were also the instructors of the course, they had direct pedagogical contact with participants during the semester. However, no privileged access beyond regular classroom interactions was established for research purposes. Participation in the study was voluntary, informed consent was obtained from all participants, and confidentiality and anonymity were ensured throughout the research process. To minimize potential power imbalances and researcher bias, participants were informed that their decision to participate or not would have no influence on course assessment, and data analysis was conducted after the completion of the didactical experience.

Data collection involved a combination of complementary qualitative methods to capture the complexity of participants' experiences during the design process. Using multiple sources of evidence enabled methodological triangulation, enhancing the credibility and trustworthiness of the findings [40,41], allowing researchers to analyze participants' actions, decisions, and reflections from multiple perspectives. The data sources included classroom observations, records and sketches from the participants' notebooks, reflective journals, artifacts (3D-printed manipulatives), and visual records (photographs and Tinkercad screenshots).

Classroom observations were conducted throughout the didactical experience using unstructured observation, supported by open-ended field notes. The researchers documented naturally occurring interactions, difficulties, design decisions, technological challenges, and moments of reflection emerging during participants' engagement in the ED process. Although observational records were open-ended, attention was guided by the research questions and the analytical framework. A central data collection instrument in this study was the reflective journal, used to document participants' experiences, decisions, and reflections throughout the ED process. Reflective journals written by participants in practical educational settings constitute an important source of narrative research, as they allow researchers to access participants' perspectives, interpretations, and experiences in context [42]. In this study, the reflective journal served both as a pedagogical tool and as a research instrument, supporting reflection during the learning process while generating rich qualitative data for analysis. Prior to producing the journals, participants recorded ideas and preliminary sketches in their notebooks during classroom sessions as they progressed through the design task. These preliminary records were subsequently incorporated into digitally created reflective journals. The journal prompts were organized according to the steps of the ED cycle and were designed to encourage reflection on both cognitive and affective aspects of learning. Specifically, participants were invited to describe the decisions they made during each step of the process, reflect on the strategies used to solve problems, express their perceptions and emotional responses to the challenges encountered, and identify possible improvements for subsequent stages of the design process. These prompts aimed to support reflective thinking and to capture evidence of how participants' understanding and knowledge evolved throughout the iterative design process.

Before moving to analysis, records, sketches, screenshots, photographs, and artifacts were systematically organized by group and by stage of the ED process. Visual materials were analyzed as complementary sources of evidence to document the evolution of participants' design trajectories and support triangulation across data sources. The analysis focused on dimensions aligned with the study framework, including the evolution of the artifact across the design stages, evidence of mathematical representations and content-related decisions, technological modeling choices and refinements, or pedagogical adaptations related to the intended classroom use of the manipulatives. Visual information was always interpreted in parallel with reflective journals and classroom observations to strengthen the credibility of interpretations and avoid isolated analysis.

The unit of analysis consisted of meaningful episodes extracted from the different data sources (reflective journals, classroom observations, sketches, screenshots, photographs, and artifacts). An episode corresponded to a coherent segment of action, reflection, or decision-making related to one or more stages of the engineering design process and was coded according to the TPACK components and the level of knowledge integration evidenced.

To analyze and systematically interpret the data, a theory-informed qualitative content analysis approach was adopted, relying on content analysis to identify patterns and relationships across multiple data sources. Content analysis is widely used in qualitative research to organize and interpret textual and visual data, allowing researchers to generate categories that reflect participants' actions and meanings [43]. Data analysis was conducted through an iterative process involving repeated reading and examination of the data, identification of meaningful units related to the research aim, coding of data segments, development of categories of analysis, triangulation of evidence from multiple data sources, and interpretation of patterns across the ED process. No qualitative data analysis software was used. All coding and categorization were conducted manually and involved both authors in iterative cycles of comparison and interpretation. While the main analytical categories (steps of the ED cycle, components of TPACK, and levels of knowledge integration, presented in Table 2) were informed by the theoretical framework and research questions, inductive analysis enabled the identification of recurrent patterns and contextual nuances emerging from the data. Reflective journals, classroom observations, and written records were segmented into meaningful episodes of decision-making, problem-solving, technological modeling, pedagogical reasoning, and redesign. Visual materials were analyzed as complementary sources of evidence and interpreted alongside textual data to identify patterns related to the evolution of designs, mathematical representations, technological refinements, pedagogical adaptations, and knowledge integration across the stages of the ED cycle. Coding decisions and interpretations were discussed collaboratively, and discrepancies were resolved through re-examination of the data until consensus was achieved. A formal inter-coder agreement coefficient was not calculated, as the study followed an interpretative qualitative approach. Instead, analytical credibility was strengthened through collaborative coding, iterative discussion, and consensus-building between the researchers.

Table 2. Categories, subcategories, and indicators of analysis.

Category	Subcategory	Indicators
Steps of the ED cycle	Problem	Identification of the task, recognition of constraints, definition of mathematical objectives.
	Imagine	Generation of ideas and brainstorming of possible solutions.
	Design	Planning of the artifact and creation of sketches or digital models.
	(Re)Build	Construction of the digital model or physical prototype.
	(Re)Test and evaluate	Testing functionality and identifying limitations.
	Redesign	Modification of the model to improve performance.
	Solution	Final presentation and justification of the artifact.
	Content knowledge (CK)	Identification and application of mathematical concepts.
	Technological knowledge (TK)	Use of digital tools and modeling procedures.
	Pedagogical knowledge (PK)	Consideration of teaching strategies and learning objectives.

Components of TPACK	Pedagogical content knowledge (PCK)	Adaptation of mathematical concepts to teaching contexts.
	Technological content knowledge (TCK)	Representation of mathematical ideas through digital models.
	Technological pedagogical knowledge (TPK)	Use of digital tools to support learning.
	Technological pedagogical content knowledge (TPACK)	Integration of content, pedagogy, and technology in the design and use of manipulatives.
Levels of knowledge integration	Single-domain	Decisions are based on one domain of knowledge (content, pedagogy, or technology) without explicit connections to others.
	Cross-domain	Two domains of knowledge are connected (e.g., content and technology or content and pedagogy), but integration remains partial.
	Integrated	Content, pedagogy, and technology are explicitly and coherently integrated to support learning goals and instructional use.

The categories *steps of the ED cycle* and *components of TPACK* were used to identify, respectively, the steps of the design process in which participants were engaged and the types of knowledge mobilized during the development of the manipulatives. These categories provided a structural framework for organizing the data and for examining how participants' actions and decisions unfolded across the design process. The category *levels of knowledge integration* aimed to capture the degree to which different knowledge domains were connected and integrated during the design process. Therefore, analyzing the level of integration allows one to move beyond identifying isolated forms of knowledge and to examine how knowledge domains were combined in meaningful ways during the design and production of the manipulatives. The three levels of knowledge integration used in this study (single-domain, cross-domain, and integrated) should be understood as an analytical framework developed for this research, informed by the integrative nature of the TPACK framework [1,3] and by studies emphasizing the dynamic coordination of technological, pedagogical, and content knowledge in authentic teaching contexts [28]. Rather than representing a developmental model, these levels describe the extent to which different knowledge domains were coordinated within each unit of analysis. The classification of integration levels was discussed collaboratively between the researchers and validated through comparison across multiple data sources associated with the same episode of the ED process.

3.2. Didactical experience

The didactical experience was planned to engage pre-service teachers in the design and production of mathematical manipulatives through the ED process, using digital modeling tools and 3D printing technologies. Participants worked collaboratively in pairs, forming six groups, and engaged in a structured design task. This collaborative organization supported the exchange of ideas, collective problem-solving, and shared decision-making throughout the design process, characteristic of engineering design-based learning environments.

The central task required each group to *construct a 3D model of a mathematical manipulative to be used in mathematics lessons*. Participants were encouraged to create either an innovative manipulative or a redesigned version of an existing manipulative using Tinkercad and 3D printing. Each manipulative had to be accompanied by a set of two to three mathematical tasks designed to explore its educational potential and aligned with curricular expectations. Participants documented their progress throughout the process using a reflective journal, which supported systematic reflection on their learning experiences and decision-making processes.

The didactical experience combined classroom sessions with autonomous and collaborative work, allowing participants to engage in iterative cycles of design, testing, and redesign consistent with the principles of engineering design-based learning. It was organized as a sequence of interconnected stages, guiding participants from theoretical preparation to the evaluation of the pedagogical potential of the manipulatives developed through the ED process (Figure 3).

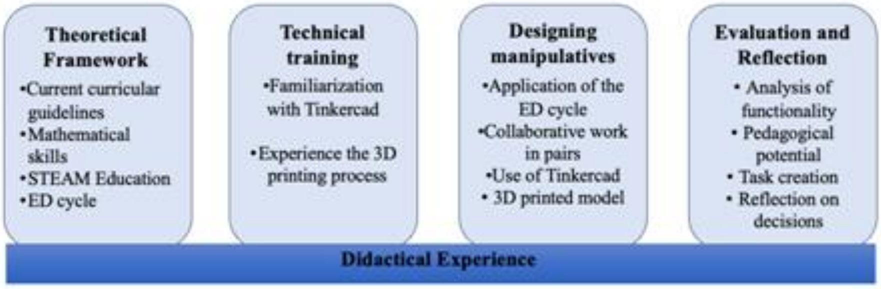

Figure 3. Structure of the didactical experience.

Throughout their training, these participants had diverse experiences framed by current curricular guidelines in mathematics education, particularly focusing on mathematical skills. Given the importance of authentic problems and interdisciplinarity, we considered STEAM education a meaningful framework to connect these dimensions, specifically highlighting the importance of problem-solving and the potential for establishing connections. Within this framework, we aimed to enable pre-service teachers to create manipulatives of their choice using digital making and to reflect on and discuss their choices through the ED process, a widely recognized approach to STEAM education, using the Vale et al. [15] model presented in Figure 2.

4. Results and discussion

4.1. Enactment of TPACK across the engineering design process

4.1.1. Problem

During the *Problem* step, participants were introduced to the central challenge of the didactical experience: to design and construct a 3D model of a mathematical manipulative using Tinkercad and 3D printing, accompanied by a set of mathematical tasks designed to explore its educational potential. Each group was required to either create an innovative manipulative or propose a reformulated version of an existing one, ensuring its suitability for classroom use and alignment with curricular goals.

Within this structured framework, participants began by selecting a manipulative resource and defining its intended educational purpose. This initial step required them to analyze the characteristics and functions of potential materials, consider their pedagogical relevance, and evaluate their feasibility for digital modeling and production. The process was therefore not limited to identifying a mathematical topic but also involved making informed design decisions grounded in curricular, pedagogical, and technological considerations.

To provide a clear overview of the design decisions made during this stage, each group selected a specific manipulative to redesign or create using digital modeling and 3D printing technologies. The selected manipulatives reflected different mathematical focuses and pedagogical intentions, ranging from the reformulation of traditional resources to the development of game-based materials. Table 3 summarizes the manipulatives developed by each group and the mathematical concepts associated with their use.

Table 3. Manipulatives selected by each group and associated mathematical focus.

Group	Manipulative developed	Mathematical focus	Type of resource
A	Fraction circle	Fraction equivalence; part–whole relationships	Reformulated traditional manipulative
B	Fraction game board with cards and dice	Fractions; part–whole relationships	Game-based manipulative
C	Circular sectors for angle representation	Angle measurement; comparison and decomposition of angles	Reformulated traditional manipulative
D	Board game–inspired manipulative	Logical reasoning; spatial reasoning; problem solving	Game-based manipulative
E	Domino game	Multiplication facts; numerical relationships	Game-based manipulative
F	Adapted ten-frame	Fractions; decimal representation; area relationships	Reformulated traditional manipulative

Across groups, the selection of manipulatives was strongly guided by the identification of mathematical concepts perceived as challenging for students. For example, Group A selected a fraction circle designed to support the understanding of equivalence relationships between fractions, explicitly focusing on denominators that allow meaningful comparisons between part–whole relationships. As described in their reflective journal, the material was intended to enable students to "recognize the equivalence between fractions representing half, fourth, and one-eighth of a unit". This decision illustrates the mobilization of CK, particularly in identifying key mathematical relationships and structuring them in ways that support conceptual understanding.

Similarly, Group C identified persistent difficulties in students' understanding of angle measurement as the starting point for their design decisions. Their reflective journal explicitly states that "teaching geometry, in particular the concept of angle and its measure, often proves to be challenging for students", motivating the development of a manipulative composed of circular sectors representing different angle measures. This reasoning reflects the activation of PCK, as participants connected specific mathematical difficulties with the design of a concrete resource to support conceptual learning.

In other cases, design decisions were guided by the intention to create engaging and interactive learning experiences. Group B proposed a manipulative with a gamified structure, incorporating elements such as a dice and fraction cards to promote active participation and exploration of mathematical relationships. The group emphasized that the material should be “an innovative approach to learning and the consolidation of part-whole relations”, highlighting the role of interaction and engagement in supporting mathematical understanding. This approach illustrates the mobilization of PK, particularly in the design learning environments that foster engagement and meaningful participation.

A particularly illustrative example of decision-making emerged in Group F, where participants explicitly considered multiple alternatives before selecting their final design. According to their reflective journal, two main options were initially explored: the creation of a domino-based resource integrating clock representations and the adaptation of a ten-frame manipulative. The final decision was influenced by technological constraints, particularly the limitations of the 3D printer, leading the group to discard the domino design and select the adapted ten-frame as a more feasible solution. This process demonstrates the use of TK and TCK, as participants balanced pedagogical relevance with technical feasibility.

Important evidence of analytical reasoning was also observed in Group D, where participants selected a manipulative inspired by a board game (Figure 4) based on its potential to support logical reasoning, spatial organization, and problem solving. The group explicitly justified their choice by emphasizing that the resource would “stimulate critical and strategic thinking, promote students’ active engagement, and create a motivating environment”. This decision reflects the mobilization of PCK, as these future teachers considered how the characteristics of the resource could support both mathematical reasoning and student engagement.

Figure 4. Board game selected by group D.

Across groups, the process of selecting a manipulative was rarely linear. Participants explored several possibilities before reaching a final decision, often evaluating alternatives in relation to pedagogical value, usability, and technical feasibility. For example, Group E initially considered different game-based resources, including Tetris and number-card games, before selecting a domino design that could support the learning of multiplication in a structured and interactive way, illustrating the exploratory nature of this ED step.

Another important feature of this step was the systematic search for existing resources and reference materials. Participants frequently analyzed traditional manipulatives commonly used in mathematics education, such as fraction circles, geoboards, pattern blocks, MAB, or the ten-frame, in order to understand their structure and functionality. In several cases, they also examined manipulatives previously produced by other colleagues with 3D printing technologies (Barbosa & Vale, 2025), using them as references for dimensions, thickness, and structural stability. Classroom observation indicates that groups compared their designs with previously printed artifacts to determine appropriate measurements and ensure printability.

In addition to observing physical resources, participants also consulted online platforms to gather ideas for their own designs. For example, Groups B and E conducted online searches to explore different models of domino games (Group E) and boards to work on part–whole relations (Group B) to help them define their decisions. These actions are illustrated in Figure 5.

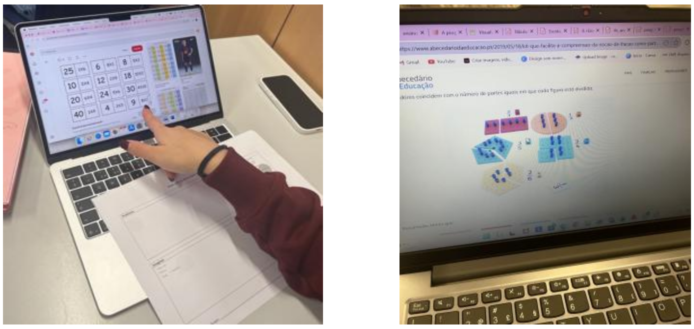

Figure 5. Search for manipulatives on online platforms.

Figure 5, therefore, provides visual evidence of the role of digital tools in supporting the exploration of design possibilities during the initial phase of the ED process. This practice illustrates the mobilization of TK, particularly in using digital tools and online resources to support design decisions.

At this stage, differences between groups began to emerge, particularly regarding the extent to which pedagogical considerations informed the selection of manipulatives, with some groups focusing primarily on mathematical or technological feasibility while others more explicitly anticipated classroom use.

Overall, the *Problem* step established the conceptual and operational foundation for the subsequent stages of the ED process. By engaging participants in the selection and justification of manipulatives through the analysis of curricular requirements, pedagogical goals, and technological constraints, this phase fostered the early mobilization of CK, PK, TK, and, in some cases, PCK and TCK. Knowledge integration during this stage remained predominantly single-domain or cross-domain, as participants typically coordinated mathematical, pedagogical, or technological considerations in partial rather than fully integrated ways. Nevertheless, these processes represent the initial enactment of TPACK, as participants began to coordinate disciplinary knowledge domains in response to a structured design challenge within a digital making environment.

4.1.2. Imagine

The *Imagine* step was characterized by the exploration of the selected manipulatives through brainstorming, observation, and data collection. Once the material was chosen, participants began to examine its possible structure, functions, and the relationships between components, gathering information to support later design decisions. Rather than defining the final plan in detail, this step focused on understanding how the material worked and how it could be adapted to the digital making context.

A central feature of this step was the systematic analysis of existing resources similar to the selected manipulatives. Participants frequently examined structured manipulatives commonly used in mathematics education to explore their characteristics and functions. This process was particularly clear in Groups A and C, who engaged in analytical practices when examining circular manipulatives.

Both groups observed structured materials and previously printed artifacts to understand how the organization of the pieces and their dimensions and thickness influenced usability. In the case of Group C, participants analyzed circular sectors representing different angle measures to determine how the pieces could represent mathematical relationships. Group A used an existing circular base to define the maximum dimensions of their fraction manipulative, ensuring that the final product would fit within the available printing area (Figure 6).

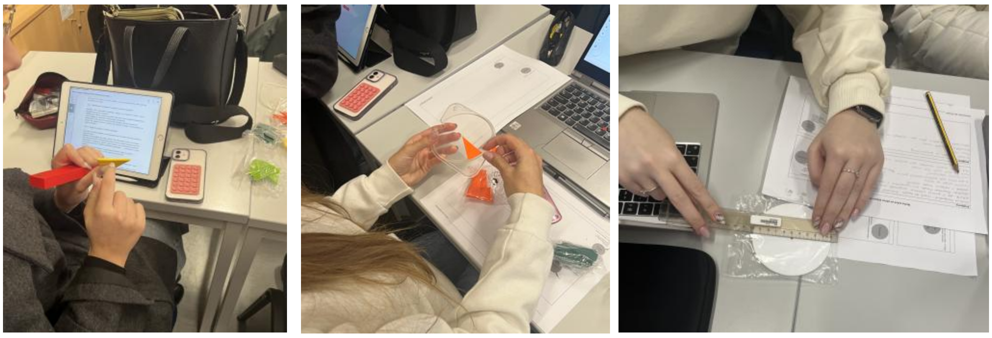

Figure 6. Groups A and C analyzing structured circular sectors.

This behavior reflects the mobilization of TCK, as participants translated mathematical representations into physical forms while considering the constraints of the production process.

In addition to analyzing dimensions, participants examined the functional characteristics and educational purposes of the manipulatives. This was particularly evident in Group D, who focused on the structure of their game-based manipulative, including its usability, number of components, and adaptability to classroom contexts. During this step, the group reviewed different versions of the game and concluded that producing the traditional number of pieces would require excessive material and time. As reported in their journal, the group emphasized that "maintaining the original number of pieces would make the production process too complex and time-consuming, so we decided to simplify the structure while preserving the educational purpose of the game". This decision illustrates the integration of CK and TK, as participants considered both mathematical relationships and production feasibility. At the same time, it reflects the activation of PK, since the group explicitly analyzed how the structure of the game would influence students' interaction and understanding. Rather than focusing solely on technical aspects, the group evaluated the material in terms of its educational functionality, demonstrating an emerging form of TPK.

Another example of exploratory reasoning can be observed in Group E, who simulated the construction of their domino manipulative using paper before moving to digital modeling. During this process, participants analyzed the content, the arrangement of the pieces, how they would connect, and the overall functioning of the game. They also consulted traditional domino sets and previously printed artifacts to determine appropriate dimensions and thickness (Figure 7).

Figure 7. Group E simulating the construction of a domino using paper and reference materials.

Similarly, Group B analyzed the dimensions of structured materials, such as a hundred plates from the MAB and a geoboard previously printed using 3D technology. These materials were used to determine the appropriate size of the board they intended to construct. This process is illustrated in Figure 8, highlighting how participants relied on existing artifacts to support design decisions. In both groups, this behavior reflects the integration of TK and CK, as participants used measurement and comparison to ensure functionality and feasibility.

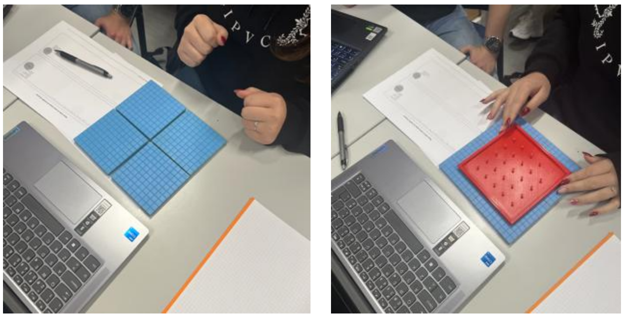

Figure 8. Group B exploring the dimensions of physical materials.

A comparable process occurred in Group F, who analyzed a ten-frame and different geometric shapes to define the structure of their manipulative. They also examined previously printed artifacts to determine the appropriate thickness of the connectors between pieces. This process is illustrated in Figure 9, showing participants using physical references to explore structural relationships before defining their design. This activity demonstrates the mobilization of TCK and TPK, as they considered how structural properties would influence the usability of the material in classroom contexts.

Figure 9. Group F exploring the dimensions of physical materials.

Although all groups engaged in exploratory reasoning, some demonstrated stronger integration by more explicitly coordinating mathematical, pedagogical, and technological considerations during the evaluation of alternative solutions, whereas others focused predominantly on the technological or structural aspects of the manipulatives.

Overall, the *Imagine* step was characterized by exploratory reasoning and systematic data collection. Through brainstorming, observation of structured manipulatives, analysis of previously printed artifacts, simulation of possible configurations, and evaluation of functional characteristics, participants developed a deeper understanding of the requirements of their chosen materials. These processes illustrate a more explicit enactment of CK, TK, and particularly TCK, as participants combined mathematical reasoning with technological awareness regarding dimensions, usability, and production constraints. Evidence of PK and PCK also emerged when future teachers considered the educational purposes and classroom use of the manipulatives. Compared with the *Problem* step, knowledge integration became more explicit, moving from predominantly single-domain and cross-domain considerations toward stronger coordination between mathematical, technological, and pedagogical knowledge in preparation for the subsequent *Design* phase.

4.1.3. Design

The *Design* step marked the transition from exploratory reasoning to the formal planning of the manipulative to be constructed. During this stage, participants began to define the structure of their artifacts more explicitly, specifying the number of components, spatial organization, dimensions, and relationships between parts. A central feature of this step was the production of sketches that served as visual representations of the intended models and as references for the subsequent construction process in Tinkercad.

Across groups, sketches played different roles and reflected varying levels of detail and planning. In some cases, sketches acted as structured design tools supporting precise decisions about measurements and organization. In others, they served mainly as conceptual representations of the material's shape.

This variation is particularly evident in Groups F and C, whose sketches were highly detailed and included explicit information about measurements, number of pieces, and structural relationships between components. As illustrated in Figure 10, these sketches show clearly defined dimensions and labeled elements, indicating that participants used drawing as a planning tool to anticipate construction requirements and ensure coherence between the mathematical structure of the material and its physical implementation. In these cases, the sketches represented a bridge between mathematical reasoning and technological production, enabling participants to visualize the final structure before initiating the digital modeling process. As stated by Group C in their reflective journal, "drawing the manipulative first helped us understand proportions before starting in Tinkercad".

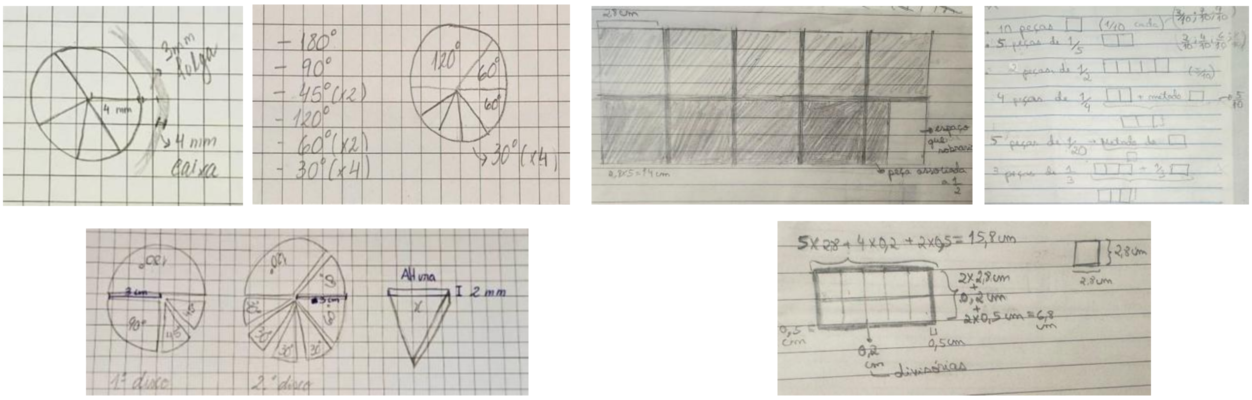

Figure 10. Clear and complete sketches produced by Groups C and F.

Such practices reflect the mobilization of TCK, as participants translated mathematical relationships into measurable design specifications. At the same time, they demonstrate the integration of CK, particularly in defining the relationships between components—for example, the partitioning of a whole into equal parts or the spatial arrangement of pieces within a structured system. The presence of numerical measurements and explicit structural planning also indicates an emerging form of TK, as participants considered the technical feasibility of their designs prior to construction.

A similar approach can be observed in Group D, whose sketches also attempted to represent the structure of the manipulative and the relationships between its components. However, these sketches were less systematic and less organized than those of Groups F and C. As shown in Figure 11, the sketches included relevant elements—such as the shape and arrangement of pieces and the general configuration of the material—but lacked precise measurements and clear visual organization. From an analytical perspective, this option reflects a partial enactment of TCK, in which participants recognized the need to represent the structure of the material but did not yet translate all mathematical and structural relationships into precise specifications.

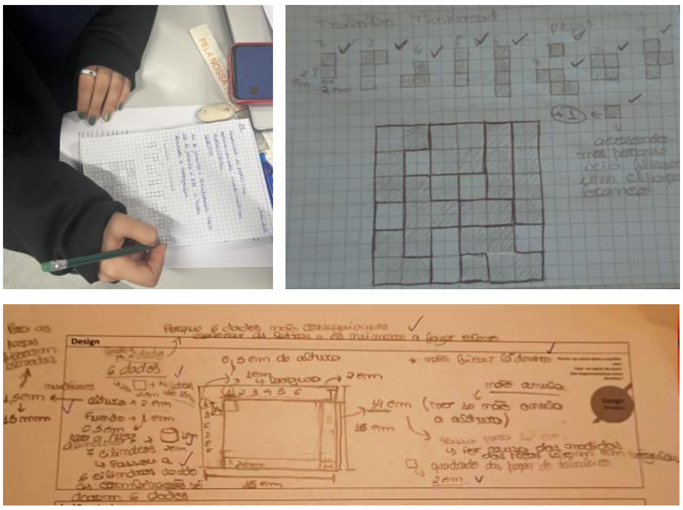

Figure 11. Sketches produced by Group D.

A different situation can be observed in Group B, whose sketch focused primarily on the general shape and structure of the material rather than on measurements or detailed planning. The sketches (Figure 12) represented the main components of the manipulative and their relative positions but did not include quantitative information about dimensions or the number of pieces. In this case, the sketch acted mainly as a conceptual representation of the intended artifact, supporting visualization rather than detailed planning.

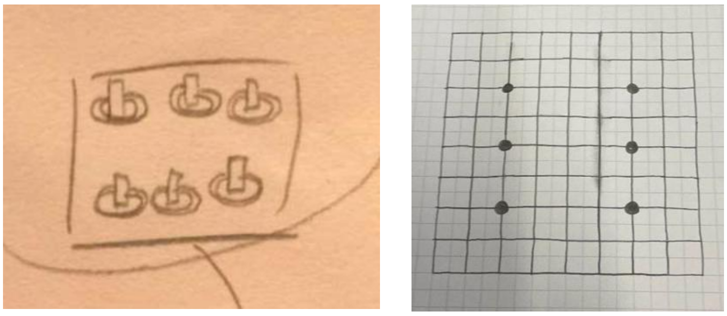

Figure 12. Incomplete sketches produced by Group B.

This behavior reflects the mobilization of CK and PK, as participants concentrated on the functional and educational aspects of the material rather than on its technical specifications. At the same time, the limited level of detail suggests that the transition from conceptual planning to technological implementation had not yet been fully articulated, indicating an early stage in the integration of technological considerations.

Not all groups engaged in sketch-based planning. Groups A and E moved directly from the *Imagine* step to the construction of the digital model in Tinkercad, without producing formal sketches beforehand. Instead of relying on paper-based planning, these participants used the digital modeling environment itself as a space for experimentation and design development.

Groups that more systematically connected sketches and design decisions to anticipated classroom use tended to demonstrate stronger integration of pedagogical and technological considerations than those whose attention remained focused primarily on technical modeling.

From a TPACK perspective, this step represents a critical moment in the ED process, as participants begin to translate conceptual ideas into concrete representations capable of guiding technological production. The act of sketching—or choosing not to sketch—became an indicator of how participants coordinated knowledge domains in preparation for construction. During this phase, evidence of CK, TK, and particularly TCK became more explicit, as participants transformed mathematical ideas into feasible design representations while considering dimensions, materials, and technological constraints. Evidence of PK and PCK also emerged when groups discussed usability and the pedagogical functions of the manipulatives for classroom learning. This growing coordination became visible in participants' reflections, which increasingly connected mathematical structure, dimensions, usability, and anticipated classroom implementation when describing their design choices. Compared with the *Imagine* step, knowledge integration became more explicit and intentional, moving toward stronger coordination between mathematical, pedagogical, and technological considerations. In this sense, the *Design* step can be understood as a transitional stage that established the foundation for the iterative processes of building, testing, and redesign that characterize the subsequent phases of the ED cycle.

4.1.4. Build

During the *Build* step, all groups began working in the Tinkercad environment to construct the components of their manipulatives and prepare them for 3D printing. A common strategy across groups was to design and print the pieces progressively, producing one piece at a time and testing their functionality before proceeding to the next element. This step-by-step approach allowed participants to monitor the feasibility of their designs and adjust dimensions or structural properties as needed. Rather than printing the entire model at once, groups chose to construct individual pieces sequentially, reducing the risk of errors and facilitating iterative refinement. This practice reflects the enactment of TK, particularly in understanding the constraints of digital fabrication and the need to adapt design decisions to the technical characteristics of the printer.

A central feature of this step was the active use of digital modeling tools within the CAD environment. Participants manipulated and modified geometric shapes, defined measurements, aligned components, adjusted scales to ensure the functionality of the printed pieces, and used text and hole functions. Figure 13 illustrates the work developed by the future teachers in Tinkercad, where they repeatedly resized, aligned, and adjusted components within the CAD environment to ensure that digital models corresponded to the intended mathematical structure and could be successfully printed.

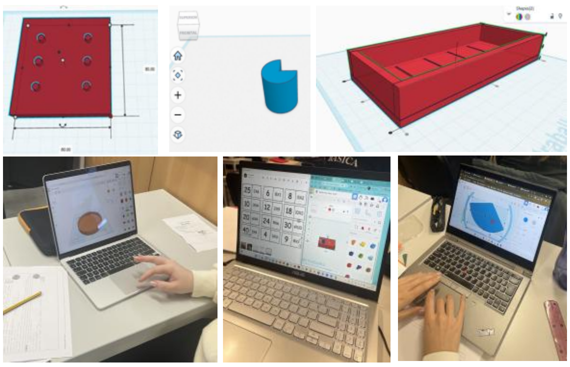

Figure 13. Building the model with Tinkercad.

This process reflects the integration of CK and TK as mathematical reasoning about measurement and scale-informed digital modeling decisions.

However, the transition from digital modeling to physical production also revealed technical challenges. One of the most frequently reported difficulties was determining appropriate dimensions and scale for the printed pieces. Participants had to consider factors such as thickness, fit, and structural resistance, often requiring adjustments to the original design. For example, Group C reported that "*the* use of Tinkercad and 3D printing proved to be both challenging and enriching. Among the main difficulties were defining appropriate dimensions, managing the scale of the model, and choosing the thickness of the pieces". These difficulties highlight the role of TCK, as participants needed to translate mathematical relationships into physical objects while accounting for material constraints.

Despite these challenges, participants generally perceived the modeling and printing process as engaging and motivating. Many described a sense of satisfaction and curiosity when observing the transformation of digital designs into physical objects. The completion of the printing process represented a significant milestone in the construction of the manipulatives. Participants observed the final printed pieces, tested their functionality, and evaluated whether the components fit together correctly. In many cases, the successful assembly of the pieces generated a strong sense of accomplishment and confidence. As described by Group C: "Seeing the manipulative completed represented the culmination of a long and demanding process that involved reflection, decision-making, and continuous learning".

Differences between groups became more visible during this phase, as some participants engaged more extensively in revising dimensions, usability, and functionality in response to emerging constraints, while others implemented fewer modifications and maintained more linear construction processes.

Overall, the *Build* step transformed abstract ideas into tangible artifacts through iterative modeling and testing. By constructing their manipulatives piece by piece and refining their designs based on real-world feedback, participants developed a deeper understanding of the relationship between digital design and physical production. During this phase, TK and TCK became particularly evident, as participants adjusted dimensions, shapes, and modeling strategies in response to technological and material constraints. Evidence of PK and PCK also emerged when groups reconsidered the usability and pedagogical effectiveness of the manipulatives for classroom application. Compared with the *Design* step, knowledge integration became more dynamic and interconnected, as participants were required to coordinate mathematical, pedagogical, and technological decisions simultaneously. In this sense, the iterative nature of the *Build* step created important opportunities for the progressive enactment of TPACK.

4.1.5. Test and Evaluate and Redesign and Build

The *(Re)Test and Evaluate* and *Redesign and Build* steps represented critical moments in the ED process, as participants began to test the functionality of their printed components and assess whether their designs met the intended structural and pedagogical requirements. During this stage, groups examined the physical artifacts produced through 3D printing, identified limitations or errors, and implemented adjustments to improve the usability and feasibility of their manipulatives. Participants observed how components fitted together, whether dimensions were appropriate, and whether the manipulatives could be used effectively in classroom contexts. These evaluations often revealed discrepancies between the digital model and the physical object, highlighting the importance of considering material properties, spatial relationships, and printing constraints throughout the design process.

Several groups identified structural issues related to the dimensions and thickness of the components. For example, Group F encountered difficulties when printing the ten-frame. As reflected in their journal, participants explained that "after printing, we realized that the base was too fragile, so we needed to rethink the thickness to make the manipulative resistant enough for classroom use" (Figure 14). After analyzing the problem, participants reformulated the dimensions, particularly the thickness of the base and the divisions, to ensure sufficient resistance and stability. This adjustment demonstrates the enactment of TCK, as participants connected geometric reasoning about thickness and support with the technical requirements of the printing process.

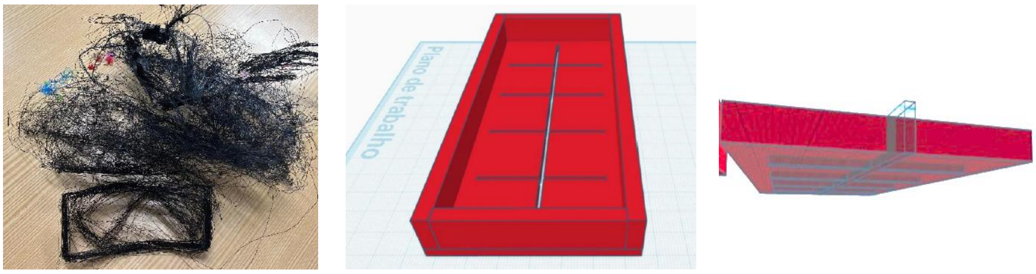

Figure 14. Evaluating and redesigning the ten-frame (Group F).

A different type of redesign occurred in Group D, where the evaluation of the printed pieces revealed spatial limitations within the structure of the manipulative. Initially, the group planned to include six cylindrical elements; however, after printing and assembling the components, they realized that there was insufficient space to accommodate all pieces. Consequently, they decided to reduce the number of cylinders from six to three, "redesigning some parts because the manipulative was not working as expected and students could become confused when using it". This decision illustrates how testing the physical prototype enabled participants to reassess the feasibility of their design and adapt it to real-world constraints. It also highlights the role of CK and TK in understanding spatial relationships and adjusting the configuration of components.

Similarly, Group C identified minor dimensional inconsistencies during the evaluation of their printed artifacts. After testing the fit between the pieces and the storage box, they decided to slightly adjust the dimensions of the box to improve functionality, reporting that "some parts would not fit inside the box as expected". Although the modification was relatively small, it reflects an important aspect of the ED process: the refinement of prototypes through careful observation and incremental improvement.

In Group B, testing revealed more significant difficulties related to the size and functionality of the connecting pieces. The group initially designed star-shaped rings with a central hole; however, after printing one of these elements, they observed that the components were too small for the printer to recognize them (Figure 15). As a result, the group reformulated the shape, using a previously developed manipulative (Tower of Hanoi disks) as a reference to determine a more appropriate element and the respective dimensions.

Figure 15. Evaluating and redesigning one of the pieces (Group B).

In Group A, the evaluation focused on the readability and functionality of the raised fraction representations on the manipulative. Although the group initially believed that the height of the fraction markings was appropriate, visual inspection of the printed model revealed that the dimensions were insufficient for effective use. After measuring the pieces more carefully, participants discovered that the height of the raised elements was smaller than required and adjusted the dimensions accordingly. They also modified the size of the box to ensure successful printing of the base. These actions demonstrate the integration of measurement, visual perception, and technical reasoning in the refinement of the design.

A more cautious strategy was adopted by Group E, who chose to print a single domino piece before producing the remaining pieces. This decision allowed them to evaluate the feasibility of the design and identify potential problems early in the process. After testing the first printed piece, the group detected issues related to the structure of the box and implemented modifications before proceeding with the full production of the manipulative. This approach reflects a deliberate use of prototyping as a method for risk reduction and iterative improvement.

Groups engaging more actively in testing and redesign tended to demonstrate stronger forms of TPACK enactment, as iterative refinement required the simultaneous coordination of mathematical, pedagogical, and technological decisions.

From a TPACK perspective, this step represents a moment of increasing knowledge integration. Participants combined mathematical understanding (CK), technological skills (TK), and pedagogical considerations (PK) to improve the functionality of their manipulatives, while evidence of TCK, TPK, and PCK became more explicit through decisions about dimensions, structure, usability, and classroom application. Compared with the *Build* step, knowledge integration became more interconnected, as disciplinary, pedagogical, and technological considerations increasingly informed decisions simultaneously rather than sequentially. These processes illustrate the emergence of stronger forms of TPACK, particularly when participants revised artifacts in response to testing outcomes and practical constraints. The process of testing and redesign thus played a central role in the enactment of TPACK, supporting professional competencies related to problem solving, critical thinking, and reflective practice.

4.1.6. Solution

In the *Solution* step, participants presented their final manipulatives, developed mathematical tasks associated with their use, and reflected on the learning experience. At this stage, the focus shifted from technical construction to pedagogical application, requiring future teachers to demonstrate how the artifacts could support mathematical learning in meaningful ways. The presentation of the final products (Figure 16) allowed for the evaluation of both the functionality of the manipulatives and their educational potential within classroom contexts. The completed manipulatives were functional and structurally coherent, reflecting the iterative process of testing and redesign carried out in the previous steps. Participants demonstrated increasing confidence in the use of digital tools and a growing awareness of the relationship between design decisions, technical constraints, and pedagogical objectives.

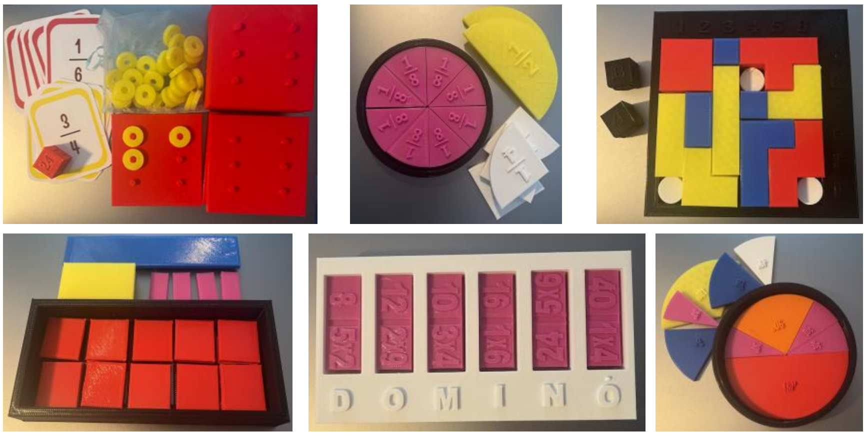

Figure 16. Artifacts produced by each group.

A central component of this step was the development of mathematical tasks intended to accompany the manipulatives. These tasks were designed to promote conceptual understanding, reasoning, and communication, while aligning the use of the materials with curricular expectations. Although all groups successfully produced tasks linked to their manipulatives, the depth of mathematical exploration and the degree to which the materials were integrated into the learning process varied across groups. Overall, the tasks addressed a range of mathematical topics commonly explored in primary education, particularly within the domains of number and operations and geometry and measurement.

Groups F and C demonstrated a more integrated use of mathematical and pedagogical knowledge. Their tasks showed strong conceptual rigor and clear progression in complexity, positioning the manipulative as a central tool for reasoning and representation. In Group F, the tasks focused on fractions, part–whole relationships, decimal representation, and area, enabling students to connect different numerical representations. In Group C, the activities explored the concept of angle through rotation, comparison, and decomposition, promoting reasoning and justification. In both cases, the manipulative functioned as a structuring element of the learning process rather than a simple visual aid.

Groups A and B developed tasks that were mathematically correct and generally aligned with curricular expectations. In Group A, the tasks focused on fraction equivalence and comparison, supporting visualization of part–whole relationships. In Group B, the tasks explored fraction concepts through a game-based context, promoting engagement and consolidation of mathematical ideas. Although the manipulatives were used consistently, the progression of cognitive demand and the depth of investigation were less explicitly developed than those observed in Groups F and C.

In contrast, the tasks proposed by Group D revealed some conceptual and pedagogical inconsistencies. While the manipulative suggested natural connections to geometric reasoning, these relationships were not fully explored in the proposed tasks. Some tasks showed limited alignment between mathematical objectives and task structure, reducing the potential of the material to support conceptual understanding.

Similarly, in Group E, the mathematical content was generally appropriate for the intended level; however, the connection between the tasks and the manipulative was not always explicit. In several situations, the tasks could be completed without directly using the material, indicating that the pedagogical role of the manipulative had not been fully integrated into the task design.

These differences suggest that stronger TPACK enactment emerged when manipulatives were treated not only as artifacts to be produced but as pedagogically meaningful tools to support mathematical learning, particularly in groups that engaged more extensively in iterative refinement.

Beyond the development of tasks, participants engaged in reflection on their experiences throughout the project. Many groups emphasized the value of transforming abstract mathematical ideas into tangible objects and recognized the importance of testing, adjustment, and persistence when working with digital technologies. The experience contributed to problem-solving skills, technological confidence, and a deeper understanding of the pedagogical potential of manipulatives in mathematics teaching.

From a TPACK perspective, the *Solution* step represents the moment at which knowledge integration became most visible. Participants demonstrated the ability to connect mathematical content (CK), technological tools (TK), and pedagogical strategies (PK) in the design of educational resources, while evidence of PCK, TCK, TPK, and integrated TPACK emerged through the alignment of artifact design with intended classroom use. Groups in which the manipulative was central to the structure of the tasks showed stronger evidence of PCK and TPACK, as the design of the artifact and tasks were closely interconnected. In contrast, groups where the manipulative played a secondary role revealed a more fragmented use of knowledge domains, characterized by the presence of technological production skills without full pedagogical integration.

4.2. Levels of knowledge integration during the design of 3D-printed manipulatives

Rather than representing fixed characteristics of specific groups, the levels of knowledge integration were observed as dynamic patterns that emerged and evolved across the iterative stages of the ED cycle. Instead of simply identifying which TPACK domains were present, the analysis examined the degree of coordination between knowledge domains during participants' design decisions. In many situations, participants initially made decisions based on a single domain of knowledge and progressively moved toward more integrated forms of reasoning as they tested, evaluated, and refined their designs. Accordingly, three analytical levels of knowledge integration were identified: *single-domain*, when decisions were guided predominantly by one knowledge domain; *cross-domain*, when two domains were explicitly coordinated; and *integrated*, when content, pedagogy, and technology were articulated simultaneously in relation to pedagogical goals and design decisions. These levels do not replace the TPACK framework or its knowledge intersections. Rather, they provide an additional analytical lens for understanding how knowledge coordination unfolded during the iterative stages of the ED process.

Evidence of *single-domain integration* was observed when decisions were guided primarily by one domain of knowledge without explicit connections to others. This pattern occurred when participants focused on solving a specific problem within one domain, responding to immediate needs or constraints. *Single-domain* decisions based on CK were observed during the initial identification of the mathematical focus of the manipulatives. For example, some groups selected specific mathematical topics—such as fractions or angles—based on perceived learning difficulties associated with these concepts. In these cases, the decision centered on the mathematical content itself, without explicit consideration of technological feasibility or instructional strategies.

Single-domain decisions based on PK were also identified when participants prioritized student engagement or classroom dynamics as the main criterion for selecting a resource. For instance, some groups justified the choice of game-based manipulatives by emphasizing their potential to motivate students and promote participation. In these cases, decisions focused primarily on pedagogical considerations, without explicit reference to mathematical structure or technological constraints.

Single-domain decisions based on TK were particularly visible during the modeling and printing stages, when participants focused on ensuring the successful production of the artifacts. For example, during the *Build* step, several groups concentrated on adjusting dimensions, thickness, or scale to ensure that the pieces could be printed correctly. These decisions were often based on technical feasibility rather than on mathematical meaning or pedagogical use. Classroom observations showed participants experimenting with measurements in the Tinkercad environment, modifying shapes, and testing printing parameters without explicit reference to learning objectives or student interaction. A clear example of this pattern can be observed in Group E, who initially focused on producing the domino pieces and ensuring that they could be printed successfully. Although the group demonstrated competence in modeling and fabrication, the relationship between the manipulative and the proposed mathematical tasks remained less explicit. In several cases, the tasks designed by this group could be completed without direct use of the manipulative, indicating that the technological production of the artifact was prioritized over its pedagogical function. *Single-domain integration* was also evident when participants responded to technical challenges without considering broader implications. For instance, when adjusting the thickness of components or modifying the size of pieces to ensure printability, some groups made decisions based solely on structural resistance or printer limitations. These decisions demonstrate engagement with technological tools but limited coordination with mathematical or pedagogical considerations.

Evidence of *cross-domain integration* was identified when participants connected two domains of knowledge—content, pedagogy, or technology—while the third domain remained implicit or less explicitly articulated. This pattern corresponds to specific forms of knowledge interaction within the TPACK framework, namely PCK, TCK, and TPK.

Several situations reflected the coordination of CK and TK, corresponding to TCK. This occurred when participants used mathematical reasoning to inform technological decisions about the design and production of the manipulatives. A representative example can be observed in Group B, who analyzed the dimensions of structured materials such as base-ten blocks and previously printed manipulatives to determine the size of the board they intended to construct. This process required the use of measurement and spatial reasoning to define dimensions that were mathematically coherent and technically feasible. Similarly, in Group A, participants adjusted the height of the fraction representations after observing that the initial dimensions were not sufficiently visible in the printed model. This decision required the integration of mathematical reasoning about measurement with technological knowledge related to printing constraints. Additional evidence of TCK was observed in the production of sketches during the *Design* step. For example, Groups C and F created detailed drawings that included measurements, the number of pieces, and structural relationships between components. These sketches demonstrate how participants translated mathematical concepts into measurable physical structures, anticipating the behavior of the design during production. Another example occurred during the testing and redesign stages. Group D reduced the number of cylindrical components after discovering that there was insufficient space to accommodate all pieces in the printed model. This decision required the integration of spatial reasoning with technological feasibility.

Evidence of PCK was observed when participants connected mathematical concepts with instructional strategies or learning objectives. For example, several groups designed mathematical tasks that required students to manipulate the materials to explore specific concepts. Group C, for instance, developed tasks that required students to compare and decompose angles, promoting reasoning and justification. Similarly, Group A designed tasks that supported the exploration of fraction equivalence through visual comparison and manipulation of the material. These situations demonstrate the coordination of mathematical understanding with pedagogical planning, even when technological considerations were not explicitly integrated into the instructional design.

Evidence of TPK was identified when participants made decisions about the design or modification of the manipulatives to support classroom use or student interaction. For example, some groups adjusted the size of pieces or the structure of the materials to improve handling, readability, or accessibility for students. These decisions were guided by considerations of usability and engagement rather than by mathematical structure alone. In other situations, participants reorganized the components of the manipulatives to facilitate manipulation during classroom activities, demonstrating awareness of how technological design could influence teaching and learning processes. These situations illustrate the connection between technological design and pedagogical intention, even when mathematical content was not the primary focus of the decision.

The highest level of knowledge integration—*Integrated*—was observed when participants explicitly coordinated content, pedagogy, and technology to support learning goals and instructional use. This level corresponds to the enactment of TPACK. Strong evidence of *integrated knowledge* was observed in Groups F and C, particularly during the *Solution* step, when participants presented their manipulatives and associated tasks. In these groups, the design of the artifact was clearly connected to specific mathematical concepts and structured learning activities that required the use of the manipulative as an essential tool for reasoning and representation. For example, Group F developed tasks that connected fractions, decimal representation, and area through the manipulation of a ten-frame model. The structure of the manipulative supported the exploration of numerical relationships, and the progression of tasks reflected increasing levels of cognitive demand. Similarly, Group C designed tasks that required students to construct and compare angles through rotation and decomposition, demonstrating a clear alignment between the design of the material, the mathematical content addressed, and the intended learning objectives.

Evidence of *integrated knowledge* was also found in participants' reflections, where several groups explicitly discussed how their design decisions were influenced by considerations of student learning. These reflections indicate awareness that the effectiveness of a manipulative depends not only on its technical functionality but also on its capacity to support conceptual understanding and classroom interaction.

Integrated knowledge was further visible when participants anticipated how students would use the manipulatives and adjusted their designs accordingly. For instance, decisions about the number of pieces, size of components, and organization of materials were sometimes guided by considerations of usability, clarity, and engagement in classroom contexts.

The different forms of knowledge integration identified in the analysis were operationalized through specific combinations of knowledge domains within the TPACK framework. Table 4 summarizes these combinations, linking each level of knowledge integration—*single-domain*, *cross-domain*, and *integrated*—to the corresponding forms of knowledge interaction (CK, PK, TK, PCK, TCK, TPK, and TPACK). The table also presents typical situations observed during the design process and illustrative examples drawn from the data, providing a structured overview of how participants mobilized and combined knowledge domains while designing and producing the 3D-printed manipulatives.

Table 4. Levels of knowledge integration and forms of knowledge interaction identified in the study.

Level of knowledge integration	Components of TPACK	Components of knowledge combined	Typical situations observed	Examples
Single-domain	CK	Content only	Selection of mathematical concepts	Choosing fractions or angles as the focus
	PK	Pedagogy only	Selection of the manipulative to increase motivation	Choosing a game-based resource
	TK	Technology only	Adjustment of dimensions to ensure printability	Modifying the thickness of pieces
Cross-domain	PCK	Content + pedagogy	Designing tasks aligned with mathematical concepts	Tasks that explore fraction equivalence
	TCK	Content + technology	Using measurements to define dimensions of the model	Adjusting the height or size of components
	TPK	Technology + pedagogy	Modifying design to improve usability	Adjusting the size of pieces for manipulation
Integrated	TPACK	Content + pedagogy + technology	Coordinating design, tasks, and instructional use	Designing manipulatives to support conceptual understanding

5. Conclusions

This study set out to understand how the ED process supports the development and integration of pre-service teachers' TPACK in digital making environments. More specifically, it aimed to examine how TPACK was enacted across the steps of the ED cycle and to identify the levels of knowledge integration that emerged during the design and production of 3D-printed manipulatives. These goals align with a growing need in teacher education research to conceptualize teacher knowledge not as a static set of competencies but as a dynamic and practice-based construct that emerges and evolves through engagement in authentic design activities and iterative problem-solving processes [1,20].

The findings provide evidence that the ED process can function as a valuable pedagogical framework for supporting the enactment and integration of professional knowledge in technology-rich learning environments. By engaging pre-service teachers in the design, production, and evaluation of mathematical manipulatives using CAD and 3D printing, the study created opportunities for participants to mobilize and connect content, pedagogy, and technology in meaningful ways. These results reinforce the view that design-based learning environments promote active knowledge construction and interdisciplinary thinking, particularly when learners engage in iterative cycles of planning, testing, and redesign [7,8,15].

Regarding the first research question, the analysis of participants' actions across the ED cycle revealed that different components of TPACK were mobilized at different stages of the design process, reflecting the dynamic and context-dependent nature of teacher knowledge. Rather than emerging simultaneously, the various forms of knowledge became increasingly integrated as participants engaged with the task, moving from exploratory and technical actions toward more pedagogically informed decisions.

In the initial stages of the process (*Problem* and *Imagine*), participants demonstrated strong engagement with CK and TK as they explored mathematical ideas, analyzed existing manipulatives, and investigated possible design solutions. These actions required spatial reasoning, measurement, and the interpretation of mathematical relationships, highlighting the role of disciplinary knowledge in guiding design decisions. At the same time, the use of digital tools and online resources reflected the mobilization of technological competence, particularly in understanding the affordances and constraints of digital making technologies.

During the *Design* and *Build* stages, participants increasingly coordinated content and technology, leading to the emergence of TCK. For example, decisions about dimensions, scaling, and structural stability required the translation of mathematical concepts into digital models, illustrating how technological tools can support the representation and transformation of disciplinary knowledge. These findings are consistent with research showing that CAD environments promote spatial visualization, iterative reasoning, and conceptual understanding through design-based activities [11,12]. However, the present study extends this literature by showing that, in teacher education contexts, CAD modeling was not limited to supporting technical or spatial reasoning. Instead, participants progressively connected modeling decisions to pedagogical intentions and anticipated classroom use, particularly when evaluating usability and designing mathematical tasks. This broader coordination of knowledge domains may be explained by the integration of the ED process with explicit pedagogical reflection, which encouraged participants to move beyond technical construction toward educational decision-making.

PK became more visible in the later stages of the process, particularly during testing, redesign, and presentation of the final solutions. Participants reflected on how their manipulatives could support student learning, considered the clarity and usability of the materials, and designed mathematical tasks aligned with curricular expectations. These actions illustrate the mobilization of PK and PCK, as participants connected mathematical concepts with instructional strategies and anticipated how learners would interact with the materials.

The highest level of knowledge coordination, TPACK, was most evident during the *Solution* stage, when participants presented their artifacts and justified their pedagogical use. At this stage, design decisions were explicitly linked to learning objectives, classroom practices, and technological functionality. These findings support the argument that TPACK is enacted and mobilized through participation in authentic professional activities that require simultaneous consideration of content, pedagogy, and technology [1,2]. This resonates with recent perspectives that conceptualize TPACK as an enacted and context-dependent form of professional knowledge rather than a stable set of competencies [19,20]. However, unlike studies that conceptualize TPACK development primarily through self-report measures or isolated technology integration tasks [27], the present study highlights how authentic design-based experiences involving digital fabrication may create opportunities for future teachers to progressively coordinate professional knowledge through situated problem solving. One possible explanation for this stronger integration is the iterative nature of the ED cycle, which repeatedly required participants to reconcile disciplinary, technological, and pedagogical considerations.

Overall, the results demonstrate that the ED process provides a structured yet flexible context for enacting and integrating teacher knowledge. The iterative nature of the process encouraged reflection, experimentation, and revision, enabling participants to deepen their understanding of both the technological tools and the pedagogical potential of the materials they created. The findings are also consistent with recent perspectives that conceptualize TPACK as an adaptive and situated form of professional knowledge in pre-service teacher education. Recent work has emphasized the importance of understanding how TPACK develops through authentic and technology-rich experiences rather than as a static set of isolated competencies [28,30]. In the present study, the progressive coordination of content, pedagogy, and technology throughout the ED cycle suggests that TPACK development emerged through iterative engagement in design tasks and reflective decision-making. Although this study does not focus on artificial intelligence, these findings also resonate with emerging discussions on intelligent TPACK, which emphasize the importance of developing integrated and context-responsive professional knowledge in increasingly complex digital environments [29].

As for the second research question, the analysis of knowledge integration revealed that participants moved across different levels of integration throughout the design process, rather than remaining at a single level. These levels—*single-domain*, *cross-domain*, and *integrated*—emerged as dynamic patterns that shifted in response to the demands of the task and the challenges encountered during design and production.

Evidence of *single-domain integration* was observed primarily when participants focused on technical or procedural aspects of the task, particularly during moments of uncertainty or technical difficulty. In these situations, decisions were guided mainly by technological considerations, such as adjusting dimensions, modifying thickness, or ensuring that components could be successfully printed. Although these actions demonstrated technological competence, they did not always involve explicit connections to mathematical meaning or pedagogical use. *Cross-domain integration* was the most frequently observed pattern throughout the study. This level was evident when participants connected mathematical reasoning with technological decision-making, such as analyzing the dimensions of existing manipulatives, determining appropriate scales for digital models, or modifying designs based on spatial constraints. These actions required the coordination of content and technology, illustrating how design activities can support interdisciplinary thinking and problem-solving. However, the pedagogical implications of these decisions were not always fully articulated at this stage. The highest level of integration, *integrated knowledge*, was evident when participants explicitly coordinated content, pedagogy, and technology to support learning goals. This level was particularly visible in the final stages of the process, when participants presented their manipulatives and designed tasks that required meaningful use of the materials. In these cases, the artifact functioned not merely as a physical object but as a pedagogical tool embedded within a coherent instructional strategy.

These findings align with models of interdisciplinary learning that conceptualize knowledge integration as a continuum rather than a fixed state [32]. Similar to prior studies on integrated STEM and engineering-based learning [7,9], participants moved between different forms of knowledge coordination depending on the demands of the task. However, the present study further illustrates how transitions between integration levels were often triggered by practical design constraints and iterative redesign decisions. This suggests that moments of difficulty and revision may play an especially important role in supporting deeper forms of TPACK enactment in digital making environments.

Despite the contributions of this study, several limitations should be acknowledged. First, the research was conducted with a relatively small group of participants (12 pre-service teachers) from a single teacher education program, which limits the broader transferability of the findings to other contexts. Although qualitative research aims to provide an in-depth understanding rather than statistical generalization, future studies involving larger and more diverse samples could provide additional evidence regarding the consistency of the observed patterns. Second, the analysis focused primarily on group-based processes and interactions rather than on individual trajectories of TPACK enactment. While collaborative work was central to the didactical experience and aligned with the iterative nature of the ED process, this approach limited a more detailed understanding of how individual participants mobilized and integrated knowledge domains over time. Third, the study focused on a single didactical experience implemented within a specific curricular context. The duration of the intervention, while sufficient to support the mobilization of design skills and reflective practices, may not have been long enough to capture the long-term enactment of teacher knowledge or the transfer of these competencies to real classroom settings. In addition, no follow-up data were collected after the intervention, limiting understanding of how participants may later have enacted TPACK in professional practice. Another limitation relates to the availability of technological resources and institutional support. The successful implementation of digital making activities depends on access to equipment, technical expertise, and adequate time for experimentation. Variations in these conditions across educational settings may influence the feasibility and impact of similar initiatives. Finally, because the didactical experience was conducted within a teacher education setting in which researchers also played an instructional role, some influence related to the dual researcher–teacher position cannot be entirely excluded. Nevertheless, the use of multiple data sources, including reflective journals, classroom observations, artifacts, and participant productions, contributed to strengthening the credibility of the analysis.

The findings of this study highlight the importance of integrating structured design-based experiences, such as the ED process, into teacher education programs. Engaging pre-service teachers in the design and production of educational resources using digital tools supports the enactment and integration of professional knowledge, enabling them to connect content, pedagogy, and technology in meaningful ways. The results also emphasize the value of iterative design and reflection in professional learning. Opportunities to test, evaluate, and refine solutions encourage critical thinking, persistence, and reflective decision-making, while helping future teachers understand how technological tools can support conceptual understanding and classroom practice. Furthermore, the study underscores the importance of aligning digital making activities with curricular goals and pedagogical intentions. Preparing teachers to design and adapt instructional materials using technologies such as CAD and 3D printing can strengthen their professional autonomy and capacity to respond to diverse learning needs.

Author contributions

Ana Barbosa: Conceptualization, Methodology creation, Investigation, Resources, Validation, Formal analysis, Data curation, Writing - original draft, Writing – review & editing, Supervision; Isabel Vale: Conceptualization, Methodology creation, Investigation, Resources, Validation, Formal analysis, Data curation, Writing – review & editing.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

This work was financially supported funded by Portuguese national funds through the FCT (Foundation for Science and Technology, I.P.) within the framework under the scope of the inED (Centre for Research and Innovation in Education) project under the reference UID/05198/2025 and the CIEC (Research Centre on Child Studies of the University of Minho) project under the reference UID/00317/2025.

Conflict of interest

We declare that there are no conflicts of interests.

Ethics declaration

The ethics principles for conducting research in education have been followed.

References

1. Mishra, P. and Koehler, M.J., Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record*, 2006, 108(6): 1017–1054. https://doi.org/10.1111/j.1467-9620.2006.00684.x
2. Schmidt, D., Baran, E., Thompson, A., Mishra, P., Koehler, M. and Shin, T., Technological Pedagogical Content Knowledge (TPACK): The Development and Validation of an Assessment Instrument for Preservice Teachers. *Journal of Research on Technology in Education*, 2009, 42(2): 123–149. https://doi.org/10.1080/15391523.2009.10782544
3. Koehler, M.J. and Mishra, P., What is technological pedagogical content knowledge (TPACK)? *Contemporary Issues in Technology and Teacher Education*, 2009, 9: 60–70. https://citejournal.org/volume-9/issue-1-09/general/what-is-technological-pedagogicalcontent-knowledge/
4. European Comission, *A STEM Education Strategic Plan: skills for competitiveness and innovation*. 2025, Comission to the European Parliament, the Council, the European Economic and Social Committee and the Committee of the Regions, Brussels, Belgium.
5. National Science Foundation (NSF), *STEM education for the future*. 2020, Subcommitee of the Advisory Committee of the Education & Human Resources Directorate, Alexandria, USA.

6. Organisation for Economic Co-operation and Development (OECD), *Future of education and skills 2030: OECD learning compass 2030*, OECD Publishing, 2019.
7. English, L.D., STEM education K-12: perspectives on integration. *International Journal of STEM Education*, 2016, 3(3): 1–8. https://doi.org/10.1186/s40594-016-0036-1
8. Kelley, T.R. and Knowles, J.G., A conceptual framework for integrated STEM education. *International Journal of STEM Education*, 2016, 3: 11. https://doi.org/10.1186/s40594-016-0046-z
9. Roehrig, G.H., Dare, E.A., Ellis, J.A. and Ring-Whalen, E., Beyond the basics: a detailed conceptual framework of integrated STEM. *Disciplinary and Interdisciplinary Science Education Research*, 2021, 3(11): 1–18. https://doi.org/10.1186/s43031-021-00041-y
10. Blikstein, P., Digital fabrication and 'making' in education: The democratization of invention, in *FabLabs: Of machines, makers and inventors*, J. Walter-Herrmann & C. Büching, Eds, 2013, 1–21. Transcript Publishers.
11. Ng, O.L. and Ye, H., Mathematics learning as embodied making: Primary students' investigation of 3D geometry with handheld 3D printing technology. *Asia Pacific Education Review*, 2022, 23(2): 311–323. https://doi.org/10.1007/s12564-022-09755-8
12. Tejera, M., Ortiz-Laso, Z., Ulbrich, E. and Lavicza, Z., 3D modelling and printing in mathematics education: Enhancing preservice teachers' knowledge through task design. In *Proceedings of the 17th ERME Topic Conference MEDA4*, 2024, 452-453.
13. English, L.D., Advancing elementary and middle school STEM education. *International Journal of STEM Education*, 2021, 8(1): 1–17. https://doi.org/10.1007/s10763-017-9802-x
14. Hester, K. and Cunningham, C., Engineering is Elementary: An Engineering and Technology Curriculum for Children. *Proceedings of the ASSE Annual Conference and Exposition*, 2007.
15. Vale, I., Barbosa, A., Peixoto, A., and Fernandes, F., Solving Problems through Engineering Design: an exploratory study with pre-service teachers. *Education Sciences*, 2022, 12(12): 1–26. https://doi.org/10.3390/educsci12120889
16. Barbosa, A. and Vale, I., Rebuilding manipulatives through digital making in teacher education. *STEM Education*, 2025, 5(4): 515–545. https://doi.org/10.3934/steme.2025025
17. Lin, K.Y., Wu, Y.T., Hsu, Y.S. and Williams, P.J., Effects of infusing the engineering design process into STEM project-based learning to develop preservice technology teachers' engineering design thinking. *International Journal of STEM Education*, 2021, 8(1): 1–15. https://doi.org/10.1186/s40594-020-00258-9
18. Barbosa, A., Vale, I. and Alvarenga, D., The use of Tinkercad and 3D printing in interdisciplinary STEAM education: A focus on engineering design. *STEM Education*, 2024, 4(3): 222–246. https://doi.org/10.3934/steme.2024014
19. Petko, D., Mishra, P. and Koehler, M., TPACK in context: An updated model. *Computers and Education Open*, 2025, 8: 100244. https://doi.org/10.1016/j.caeo.2025.100244
20. Schmid, M., Brianza, E. and Petko, D., Running in circles: A systematic review of reviews on technological pedagogical content knowledge (TPACK). *Computers & Education*, 2024, 214: 105024. https://doi.org/10.1016/j.compedu.2024.105024
21. Shulman, L.S., Those who understand: Knowledge growth in teaching. *Educational Researcher*, 1986, 15(2): 4–14. https://doi.org/10.3102/0013189X015002004
22. Ball, D.L., Thames, M.H. and Phelps, G., Content knowledge for teaching: What makes it

special? *Journal of Teacher Education*, 2009, 59(5): 389–407. https://doi.org/10.1177/0022487108324554
23. Shulman, L.S., Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 1987, 57(1): 1–22. http://dx.doi.org/10.17763/haer.57.1.j463w79r56455411
24. Ma, L., *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States,* 2nd ed., 2010, Routledge.
25. National Council of Teachers of Mathematics, *Principles to Actions - Ensuring Mathematical Success for All*, 2014, NCTM.
26. Hidayat, M., Zainuddin, Z. and Mazlan, N., The relationship between technological pedagogical content knowledge and belief among preservice mathematics teachers. *Acta Psychologica*, 2024, 249: 104432. https://doi.org/10.1016/j.actpsy.2024.104432
27. Li, M., Vale, C., Tan, H. and Blannin, J., A systematic review of TPACK research in primary mathematics education. *Mathematics Education Research Journal*, 2025, 37: 281–311. https://doi.org/10.1007/s13394-024-00491-3
28. Gonscherowski, P. and Rott, B., A systematic review of the literature on TPACK instruments used with pre-service teachers from 2017 to 2023 focused on selecting digital resources. *Journal of Computers in Education*, 2025, 1–38. https://doi.org/10.1007/s40692-025-00379-6
29. Celik, I., Kontkanen, S., Laru, J. and Dalyanci, A., Co-constructing adaptive lesson plans with GenAI: Pre-service teachers' Intelligent-TPACK and prompt engineering strategies. *Computers & Education*, 2026, 241: 105485. https://doi.org/10.1016/j.compedu.2025.105485
30. Reyes-Rojas, J., Díaz, B., Ruz-Reveco, C., Castro, A. and Reyes-González, D., Conceptualizing pre-service teachers' readiness for AI integration into teaching practices: An intelligent-TPACK approach. *Computers and Education Open*, 2026, 10: 100320. https://doi.org/10.1016/j.caeo.2025.100320
31. Simarro, C. and Couso, D., Engineering practices as a framework for STEM education: a proposal based on epistemic nuances. *International Jourof STEM Education*, 2021, 8: 53 https://doi.org/10.1186/s40594-021-00310-2
32. Vasquez, J., Sneider, C. and Comer, M., *STEM lesson essentials, grades 3–8: integrating science, technology, engineering, and mathematics*, 2013, Heinemann.
33. Papert, S., *Mindstorms: Children, computers, and powerful ideas*, 1980, Basic Books.
34. Khurma, O., Ali, N. and Khine, M., Exploring the impact of nal 3D printing integration on STEM attitudes in elementary schools. *Contemporary Educational Technology*, 2023, 15(4): 458. https://doi.org/10.30935/cedtech/13568
35. Läufer, T. and Ludwig, M., "Building" knowledge by creating manipulatives with the 3D printer: A course for mathematics student teachers. In *Proceedings of the 9th International Conference on Higher Education Advances (HEAd'23),* 2023, 19–23.
36. Huang, W. and Wang, T., Effectiveness of a three-dimensional-printing curriculum: Developing and evaluating an elementary school design-oriented model course. *Computers & Education*, 2022, 187(1): 104553. https://doi.org/10.1016/j.compedu.2022.104553
37. Tejera, M., Galiç, S. and Lavicza, Z., 3D Modelling and Printing in Teacher Education: A Systematic Literature Review. *Journal for STEM Education Research*, 2025, 9: 1–32 https://doi.org/10.1007/s41979-025-00147-2
38. Verner, I. and Merksamer, A., Digital design and 3D printing in technology teacher education.

Procedia CIRP, 2015, 36: 182–186. http://creativecommons.org/licenses/by-nc-nd/4.0/
39. Novak, J., *3D Printing in Education*. 2022, Routledge.
40. Cohen, L., Manion, L. and Morrison, K., *Research methods in education*, 8th ed., 2018, Routledge.
41. Creswell, J.W., *Research design: Qualitative, quantitative, and mixed methods approaches*, 4th ed., 2014, Sage.
42. O'Connell, T.S. and Dyment, J.E., The case of reflective journals: Is the jury still out? *Reflective Practice*, 2011, 12(1): 47–59. https://doi.org/10.1080/14623943.2011.541093
43. Miles, M.B. and Huberman, A.M., *Qualitative Data Analysis*, 1994, Sage Publications.

Authors' biographies

Dr. Ana Barbosa is a professor of Mathematics Education at the School of Education of Instituto Politécnico de Viana do Castelo in Portugal. She has a PhD in Child Studies, in the area of Elementary Mathematics. She is a researcher at the Centre for Research & Innovation in Education (inED). Among other topics, her research interests focus on didactics of mathematics, problem solving, visualization, algebraic thinking, active learning, outdoor mathematics education, and STEAM education.

Dr. Isabel Vale is a professor of Mathematics Education at the School of Education of Instituto Politécnico de Viana do Castelo in Portugal. She has a PhD in Didactics of Mathematics. She is a researcher at the Research Centre on Child Studies (CIEC-UM). Among other topics, her research interests focus on didactics of mathematics, in particular, problem solving—patterns, creativity, visualization, connections in mathematics education, and teacher training. More recently, she is interested in the design of tasks and teaching strategies in diverse contexts that are more favorable to active learning of mathematics, such as STEAM education and learning outside the classroom.

AIMS Press

©2026 the Author(s), licensee by AIMS Press. This is an open access article distributed under the terms of the Creative Commons Attribution License (https://creativecommons.org/licenses/by/4.0/).